LUISS Guido Carli
Bachelor Degree in Economics and Business

Microeconomics
Exercise Sessions

Prof. Lorenzo Ferrari

Handout by
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Microeconomics
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Exercise 1

Carlos has a utility function that depends on the number of musicals and the number of operas seen each month. His utility function is given by $U = xy^2$, where x is the number of movies seen per month and y is the number of operas seen per month. The corresponding marginal utilities are given by: $MU_x = y^2$ and $MU_y = 2xy$

a) Does Carlos believe that more is better for each good?

b) Does Carlos have a diminishing marginal utility for each good?

Solution

a) Yes. When we say that *a consumer believes that more is better for a good*, we are referring to a utility function which is increasing in the variable relative to that good, indeed it has a positive marginal utility. In this case, Carlos' utility function is increasing in both x and y, both MU_x and MU_y are positive (for every x and y larger than 0).

b) No. For what it concerns MU_x, it doesn't increase, neither decreases as x increases, it remains constant. While for MU_y, it increases as y increases. So, Carlos has a constant marginal utility for movies and an increasing marginal utility for operas.

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Exercise 2

For the following sets of goods draw two indifference curves, U_1 and U_2, with $U_2 > U_1$. Draw each graph placing the amount of the first good on the horizontal axis.

a) Sausages and chili (the consumer likes both and has a diminishing marginal rate of substitution of sausages for chili)

b) Sugar and artificial sweetener (the consumer likes both and will accept an ounce of artificial sweetener or an ounce of sugar with equal satisfaction)

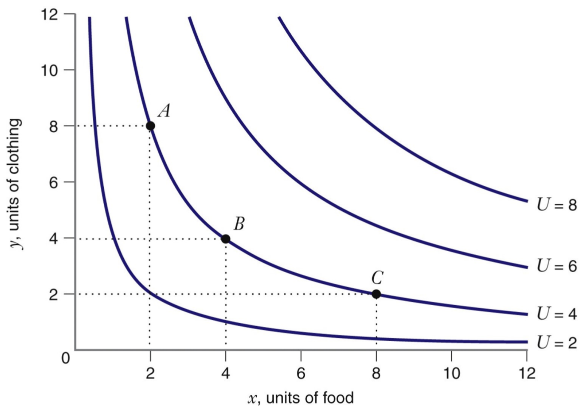
c) Nutella and jelly (the consumer likes exactly 2 ounces of Nutella for every ounce of jelly)

d) Nuts (which the consumer neither likes nor dislikes) and ice cream (which the consumer likes)

e) Apples (which the consumer likes) and liver (which the consumer dislikes)

Solution

a) **Sausages and chili**: These two goods can be represented by a Cobb-Douglas utility function.



b) **Sugar and artificial sweetener**: These two goods are an example of perfect substitutes.

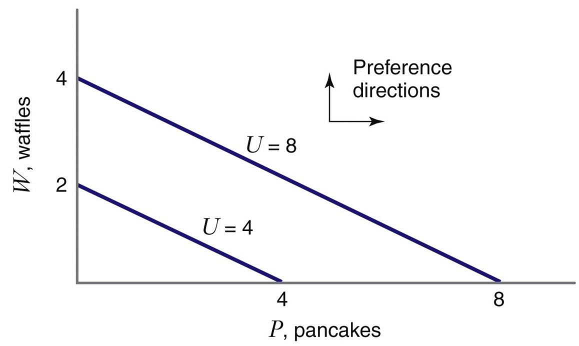
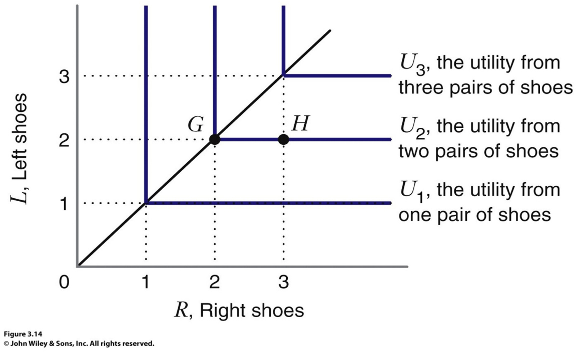
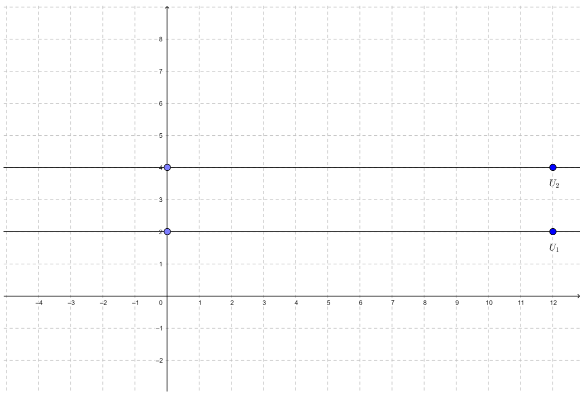


Figure 3.13

c) **Nutella and jelly**: These two goods are an example of perfect complements.



d) **Nuts and ice cream**: These two goods can be represented as an extreme case of a quasi-linear utility function. In which the indifferent curves are horizontal straight lines.



e) **Apples and liver**: In this case, apples are a good, while liver is a *bad* (a *good* for which consumer believes the more is worse)

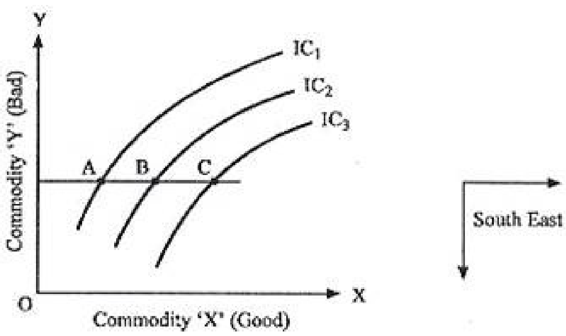


Fig. 5.15

Exercise 3

Julie and Toni consume two goods with the following utility functions:

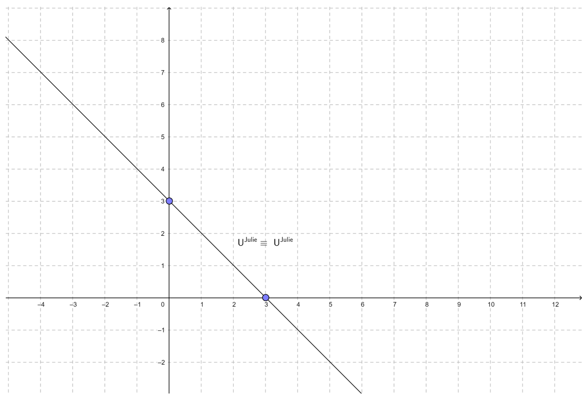
$$U^{Julie} = (x+y)^2;\ MU_x^{Julie} = 2(x+y);\ MU_y^{Julie} = 2(x+y)$$
$$U^{Toni} = x+y;\ MU_x^{Toni} = 1;\ MU_y^{Toni} = 1$$

a) Graph an indifference curve for each of these utility functions.

b) Do Julie and Toni have the same ordinal ranking of different baskets of x and y? Explain.

Solution

a) **Graph**



b) Yes. In order for Julie and Toni to have the same ordinal ranking of different baskets of x and y, taken a certain basket A which is preferred to basket B by one of the functions, it is also preferred by the other one. The two utility functions are the same, one is an increasing monotonic transformation of the other. Indeed, for every baskets A and B, if the utility of Julie relative to basket A is larger than the one of basket B, than the relation will be maintained also for Toni.

Exercise 4

Consider the utility function $U(x, y) = 3x + y$, with $MU_x = 3$ and $MU_y = 1$.

a) Is the assumption that more is better satisfied for both goods?

b) Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x?

c) What is $MRS_{x,y}$?

d) Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

e) On a graph with x on the horizontal axis and y on the vertical axis, draw a typical indifference curve (it need not be exactly to scale, but it needs to reflect accurately whether there is a diminishing $MRS_{x,y}$). Also indicate on your graph whether the indifference curve will intersect either or both axes. Label the curve U_1.

f) On the same graph draw a second indifference curve U_2, with $U_2 > U_1$.

Solution

a) Yes, because the utility function is increasing in both variables, indeed the marginal utility for x and y are always positive.

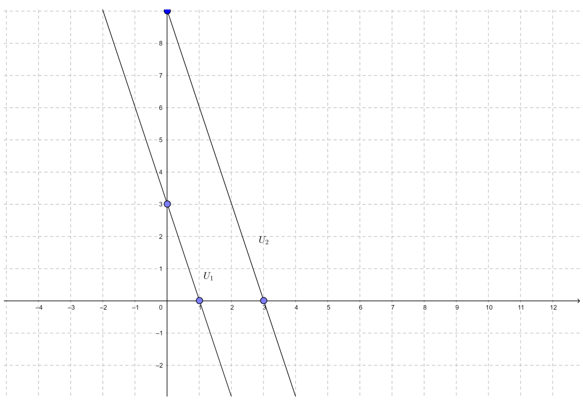
b) It remains constant, as the consumer buys more x.

c) $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{3}{1} = 3$

d) It remains constant, as the consumer substitutes x for y along an indifference curve.

e) **Graph**

f) **Graph**



Microeconomics
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February 23, 2021

Exercise 1

Suppose a consumer's preferences for two goods can be represented by the Cobb-Douglas utility function $U = Ax^{\alpha}y^{\beta}$, where A, α, and β are positive constants. The marginal utilities are $MU_x = \alpha Ax^{\alpha-1}y^{\beta}$ and $MU_y = \beta Ax^{\alpha}y^{\beta-1}$.

a) Is the assumption that more is better satisfied for both goods?

b) Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x?

c) What is $MRS_{x,y}$?

d) Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

e) On a graph with x on the horizontal axis and y on the vertical axis, draw a typical indifference curve (it need not be exactly to scale, but it needs to reflect accurately whether there is a diminishing $MRS_{x,y}$). Also indicate on your graph whether the indifference curve will intersect either or both axes. Label the curve U_1.

f) On the same graph draw a second indifference curve U_2, with $U_2 > U_1$.

Solution

a) Yes, because the utility function is increasing in both variables, indeed the marginal utility for x and y are always positive.

b) In this case, we do not know the value of α, we only know that it is positive. So, we need to study the marginal utility and to specify three possible cases:

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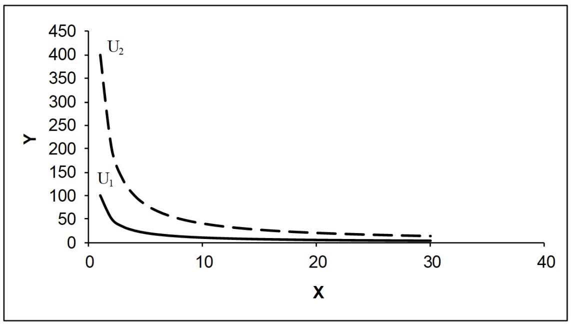
- When $\alpha < 1$, the marginal utility of x diminishes as x increases.
- When $\alpha = 1$, the marginal utility of x remains constant as x increases.
- When $\alpha > 1$, the marginal utility of x increases as x increases.

c) $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\alpha A x^{\alpha-1} y^{\beta}}{\beta A x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta}\frac{y}{x}$

d) It diminishes constant, as the consumer substitutes x for y along an indifference curve.

e) The graph below depicts indifference curves for the case where $A = 1$ and $\alpha = \beta = 0.5$.



f) See graph above.

Exercise 2

Pedro is a college student who receives a monthly salary from his parents of \$1000. He uses this salary to pay rent for housing and to go to the movies. In Pedro's town, each square foot of rental housing costs \$2 per month. Each movie he attends costs \$10. Let x denote the square feet of housing, and y denote the number of movies.

a) What is the expression for Pedro's budget constraint?

b) What is the maximum number of square feet of housing he can purchase given his income?

c) What is the maximum number of movie tickets he could attend given his income?

d) Draw a graph of Pedro's budget line.

e) Suppose Pedro's parents increase his stipend by 10%. At the same time, suppose that in the college town he lives in, all prices, including housing rental rates and movie ticket prices, increase by 10%. What happens to the graph of Pedros budget line?

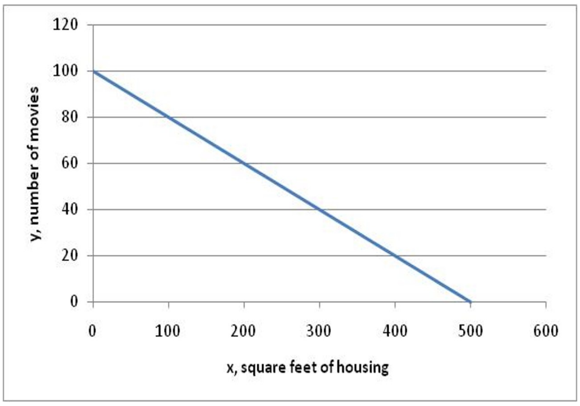
Solution

a) In order to build Pedro's budget constraint, we have to set the sum of goods weight by their relative prices smaller or equal to the maximum that Pedro can spend, namely his entire finances. Pedro's budget constraint is: $2x + 10y \leq 1000$

b) \$1000/\$2 per square foot = 500 square feet

c) \$1000/\$10 per ticket = 100 tickets

d) **Graph**



e) If everything increases by 10%, nothing changes in Pedros budget line graph.

Exercise 3

The utility that Ann receives by consuming food F and clothing C is given by $U(F, C) = FC + F$. The marginal utilities of food and clothing are $MU_F = C + 1$ and $MU_C = F$. Food costs \$1 a unit, and clothing costs \$2 a unit. Ann's income is \$22.

a) Ann is currently spending all of her income. She is buying 8 units of food. How many units of clothing is she consuming?

b) Graph her budget line. Place the number of units of clothing on the vertical axis and the number of units of food on the horizontal axis. Plot her current consumption basket.

c) Find the utility-maximizing choice of food and clothing.

Solution

a) If Ann is currently spending all of her income, this is the equation of her budget line:

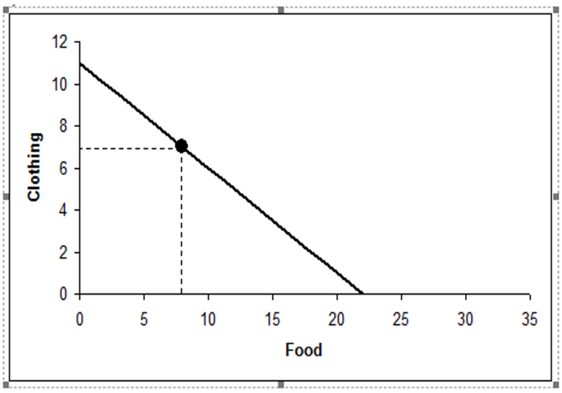
$$F + 2C = 22$$

Since she is buying 8 units of food, we have to substitute $F = 8$, then:

$$8 + 2C = 22$$

$$2C = 14$$
$$C = 7$$

b) **Graph**



c)

$$\begin{cases} MRS_{F,C} = \frac{MU_F}{MU_C} = \frac{P_F}{P_C} \rightarrow \text{tangency condition (interior solution)} \\ P_F F + P_C C = I \rightarrow \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{C+1}{F} = \frac{1}{2} = \rightarrow 2C + 2 = F \\ F + 2C = 22 \end{cases}$$

$$2C + 2 + 2C = 22 \rightarrow 4C = 20 \rightarrow C^* = 5$$
$$F = 2C + 2 \rightarrow F^* = 12$$
$$\text{Optimal Consumption Bundle } (F^*, C^*) = (12, 5)$$

Exercise 4

Eric purchases food (measured by x) and clothing (measured by y) and has the utility function $U(x, y) = xy$. His marginal utilities are $MU_x = y$ and $MU_y = x$. He has a monthly income of \$800. The price of food is $P_x = \$20$, and the price of clothing is $P_y = \$40$.
Find Eric's optimal consumption bundle.

Solution

$$\begin{cases} MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \rightarrow \text{tangency condition (interior solution)} \\ P_x x + P_y y = I \rightarrow \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{y}{x} = \frac{20}{40} = \frac{1}{2} \rightarrow 2y = x \\ 20x + 40y = 800 \end{cases}$$

$$40y + 40y = 800 \rightarrow 80y = 800 \rightarrow y^* = 10$$
$$2y = x \rightarrow x^* = 20$$
$$\text{Optimal Consumption Bundle } (x^*, y^*) = (20, 10)$$

Microeconomics
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March 2, 2021

Exercise 1

David is considering his purchases of food x and clothing y. He has the utility function $U(x, y) = xy + 10x$, with marginal utilities $MU_x = y + 10$ and $MU_y = x$. His income is $I = 10$. He faces a price of food $P_x = \$1$ and a price of clothing $P_y = \$2$.
What is David's optimal basket?

Solution

$$\begin{cases} MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \rightarrow \text{tangency condition (interior solution)} \\ P_x x + P_y y = I \rightarrow \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{y+10}{x} = \frac{1}{2} \rightarrow 2y + 20 = x \rightarrow 2y = x - 20 \\ x + 2y = 10 \end{cases}$$

$$x + x - 20 = 10 \rightarrow 2x = 30 \rightarrow x^* = 15$$

$$2y = x - 20 \rightarrow 2y = 15 - 20 \rightarrow y^* = -\frac{5}{2}$$

This result cannot be accepted (remeber $x \geq 0$, $y \geq 0$), it means it is not possible to have an interior solution, then it will be a corner solution (either $x^* = 0$ and $y^* > 0$, or $x^* > 0$ and $y^* = 0$).
Let us suppose $x^* = 0$, it follows then:

$$x + 2y = 10 \rightarrow y^* = 5 \rightarrow U(x^*, y^*) = (0)(5) + 10(0) = 0$$

Let us now suppose $y^* = 0$, it follows then:

$$x + 2y = 10 \rightarrow x^* = 10 \rightarrow U(x^*, y^*) = (10)(0) + 10(10) = 100$$

We understand that the Optimal Consumption Bundle $(x^*, y^*) = (10, 0)$

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Exercise 2

Sara views chocolate and vanilla ice cream as perfect substitutes. She likes both and is always willing to trade one scoop of chocolate for two scoops of vanilla ice cream. In other words, her marginal utility for chocolate is twice as large as her marginal utility for vanilla. Thus, $MRS_{C,V} = \frac{MU_C}{MU_V} = 2$. If the price of a scoop of chocolate ice cream (P_C) is three times the price of vanilla (P_V), will Sara buy both types of ice cream? If not, which will she buy?

Solution

Sara will buy both types of ice cream in presence of an interior solution, meaning if the tangency condition is confirmed:

$$\frac{MU_C}{MU_V} = \frac{P_C}{P_V} \rightarrow 2 \neq 3 \rightarrow \text{Corner Solution}$$

Given the MUs and the ice creams prices, it is not possible for Sara to buy both types of ice cream. We need now to understand which of the two types of ice cream she is going to buy.

$$\frac{MU_C}{MU_V} = \frac{P_C}{P_V} \rightarrow \frac{MU_C}{P_C} = \frac{MU_V}{P_V}$$

$$\frac{MU_C}{MU_V} > \frac{P_C}{P_V} \rightarrow \frac{MU_C}{P_C} > \frac{MU_V}{P_V} \rightarrow \text{Optimal Consumption Bundle } (C^*, V^*) = (C, 0)$$

$$\frac{MU_C}{MU_V} < \frac{P_C}{P_V} \rightarrow \frac{MU_C}{P_C} < \frac{MU_V}{P_V} \rightarrow \text{Optimal Consumption Bundle } (C^*, V^*) = (0, V)$$

In our case Sara is going to buy only Vanilla ice cream.

Exercise 3

Consider a consumer with utily function $U(x, y) = min(3x, 5y)$ that is, the two goods are perfect complements in the ratio $3 : 5$. The prices of the two goods are $P_x = \$5$ and $P_y = \$10$, and the consumer's income is \$220. Determine the optimum consumption basket.

Solution

As for the previous exercises, we have to set the maximization problem as a system of equation in which we have the tangency condition and the budget constraint:

$$\begin{cases} \text{tangency condition} \\ \text{budget constraint} \end{cases}$$

Differently from before, the tangency condition has to be formulated differently. Perfect complements imply that $3x = 5y \rightarrow y = \frac{3}{5}x$, and that is the tangency condition. While the budget line is $5x + 10y = 220$. Having these two elements we are ready to solve the otpimization problem.

$$\begin{cases} 3x = 5y \rightarrow y = \frac{3}{5}x \\ 5x + 10y = 220 \end{cases}$$

$$5x + \frac{30}{5}x = 220 \rightarrow 11x = 220 \rightarrow x^* = 20$$

$$y = \frac{3}{5}x \rightarrow y = \frac{3}{5}20 \rightarrow y^* = 12$$

$$\text{Optimal Consumption Bundle } (x^*, y^*) = (20, 12)$$

Exercise 4

David has a quasilinear utility function of the form $U(x, y) = \sqrt{x} + y$ with associated marginal utility functions $MU_x = \frac{1}{2\sqrt{x}}$ and $MU_y = 1$.

a) Derive David's demand curve for x as a function of the prices, P_x and P_y. Verify that the demand for x is independent of the level of income at an interior optimum.

b) Derive David's demand curve for y. Is y a normal good? What happens to the demand for y as P_x increases?

Solution

a) Denoting the level of income by I, the budget constraint implies that $p_x x + p_y y = I$ and the tangency condition is $\frac{1}{2\sqrt{x}} = \frac{p_x}{p_y}$, which means that $x = \frac{p_y^2}{4p_x^2}$. The demand for x does not depend on the level of income.

b) From the budget constraint, the demand curve for y is , $y = \frac{I - p_x x}{p_y} = \frac{I}{p_y} - \frac{p_y}{4p_x}$. You can see that the demand for y increases with an increase in the level of income, indicating that y is a normal good. Moreover, when the price of x goes up, the demand for y increases as well.

Extra Exercise

The utility that Corey obtains by consuming hamburgers H and hot dogs S is given by $U(H, S) = \sqrt{H} + \sqrt{S + 4}$. The marginal utility of hamburgers is $\frac{0.5}{\sqrt{H}}$ and the marginal utility of steaks is equal to $\frac{0.5}{\sqrt{S+4}}$.

a) Sketch the indifference curve corresponding to the utility level $U = 12$.

b) Suppose that the price of hamburgers is \$1 per hamburger, and the price of steak is \$8 per steak. Moreover, suppose that Corey can spend \$100 per month on these two foods. Sketch Corey's budget line for hamburgers and steak given this budget.

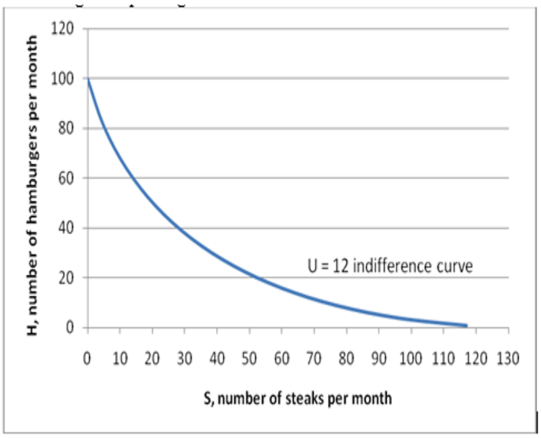
c) Based on your answer to parts (a) and (b), what is Corey's optimal consumption basket given his budget?

Solution

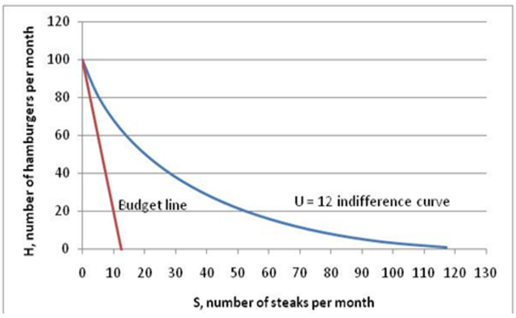
a) In order to draw the graph of this particular indifference curve, we have to set our utility function equal to 12, and to find some points on the $U = 12$ indifference curve.

S	H	U
0	100	12
5	81	12
12	64	12
21	49	12
32	36	12
45	25	12
60	16	12

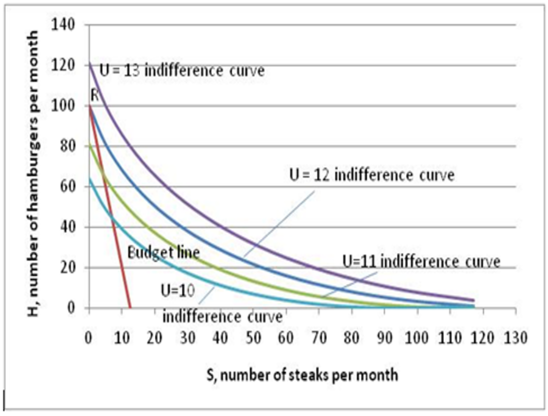
Connecting these points gives us the $U = 12$ indifference curve.



b) The equation of the budget line is $H + 8S = 100$. Graphing this on the same graph as the $U = 12$ indifference curve gives us:



c) The optimal consumption basket is $S = 0$, $H = 100$, i.e., point R in the figure below. There are several ways to see this. One way is to sketch a few more indifference curves (each corresponding to a different level of utility). This picture strongly suggests that the point of maximum utility occurs at point R.


U = 13 indifference curve
U = 12 indifference curve
U=11 indifference curve
U=10
indifference curve
Budget line
R
H, number of hamburgers per month
S, number of steaks per month
0
20
40
60
80
100
120
140
0 10 20 30 40 50 60 70 80 90 100 110 120 130

Microeconomics
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March 10, 2021

Exercise 1

Ginger's utility function is $U(x, y) = x^2y$, with associated marginal utility functions $MU_x = 2xy$ and $MU_y = x^2$. She has income $I = 240$ and faces prices $P_x = \$8$ and $Py = \$2$

a) Determine Ginger's optimal basket given these prices and her income.

b) If the price of y increases to \$8 and Ginger's income is unchanged, what must the price of x fall to in order for her to be exactly as well off as before the change in P_y?

Solution

a) In order to find the Optimal Bundle, we solve the usual system:

$$\begin{cases} MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y} \rightarrow \text{tangency condition (interior solution)} \\ P_x x + P_y y = I \rightarrow \text{budget constraint} \end{cases}$$

$$\begin{cases} \frac{2y}{x} = \frac{8}{2} = 4 \rightarrow 2y = 4x \\ 8x + 2y = 240 \rightarrow 8x + 4x = 240 \rightarrow x^* = \frac{240}{12} = 20 \end{cases}$$

$$y = 2x \rightarrow y^* = 40$$

$$\text{Optimal Bundle: } (x^*, y^*) = (20, 40)$$

b) Previously Ginger, optimizing her cosumption, reached a utility level of:

$$U(x^*, y^*) = (20)^2 40 = 16000$$

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Now, we know $p_y = 8$ and we need to calculate p_x such that, with the new prices, Ginger reaches exactly the same indifference curve as before. The new optimal bundle (x^*, y^*) must be such that: $p_x x + 8y = 240$, and $x^2 y = 16000$. We then need to add a condition to our system, since we are also looking for a third variable, and every equation in the system allow us to find a variable.

$$\begin{cases} \frac{2y}{x} = \frac{p_x}{8} = 4 \to p_x x = 16y \\ p_x x + 8y = 240 \to 16y + 8y = 240 \to y^* = 10 \\ x^2 y = 16000 \to x^2(10) = 16000 \to x^* = 40 \end{cases}$$

$$p_x x = 16y \to p_x(40) = 16(10) \to p_x^* = \frac{16(10)}{40}$$

$$\begin{cases} p_x^* = 4 \\ y^* = 10 \\ x^* = 40 \end{cases}$$

Ginger would need the price of x to decrease to \$4 in order to be exactly as well off as before.

Exercise 2

Carina buys two goods, food F and clothing C, with the utility function $U = FC + F$. Her marginal utility of food is $MU_F = C + 1$ and her marginal utility of clothing is $MU_C = F$. She has an income of 20. The price of clothing is 4

a) Derive the equation representing Carina's demand for food, and draw this demand curve for prices of food ranging between 1 and 6.

b) Calculate the income and substitution effects on Carina's consumption of food when the price of food rises from 1 to 4, and draw a graph illustrating these effects. Your graph need not be exactly to scale, but it should be consistent with the data.

Solution

a) In order to find the demand for food, we solve the usual system:

$$\begin{cases} MRS_{C,F} = \frac{MU_F}{MU_C} = \frac{P_F}{P_C} \to \text{tangency condition (interior solution)} \\ P_F F + P_C C = I \to \text{budget constraint} \end{cases}$$

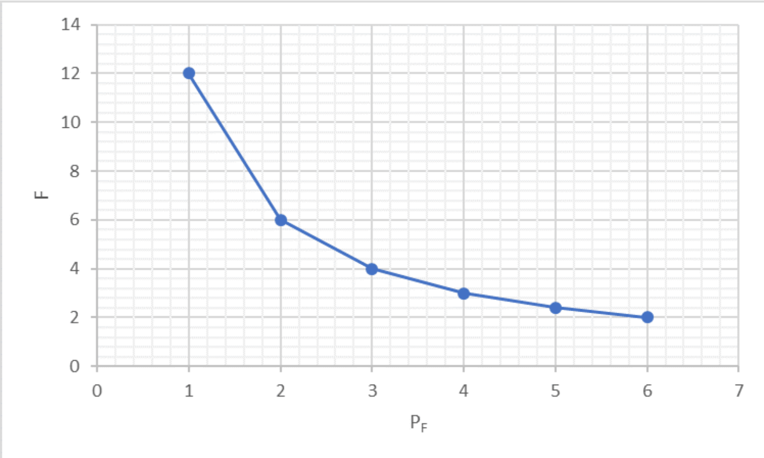
$$\begin{cases} \frac{C+1}{F} = \frac{P_F}{4} \to 4C + 4 = P_F F \to 4C = P_F F - 4 \\ P_F F + 4C = 20 \to P_F F + (P_F F - 4) = 20 \to 2P_F F = 24 \end{cases}$$

$$F = \frac{12}{P_F}$$

If we want to know also the demand for clothing, it is enough to continue solving the system:

$$4C = P_F \frac{12}{P_F} - 4 \rightarrow C = \frac{12-4}{4}$$

$$C = 2 \text{ (independently of the level of } P_F)$$



b) **Initial Basket** From the demand for food in (a) $F = \frac{12}{1} = 12$, and $C = 2$. Also the initial level of utility is $U = FC + F = 12(2) + 12 = 36$.
Final Basket From the demand for food in (a), $F = \frac{12}{4} = 3$, and $C = 2$. Also, $U = 3(2) + 3 = 9$.
Decomposition Basket Must be on initial indifference curve, with:

$$U = FC + F = 36$$

Tangency condition satisfied with final price:

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C} \rightarrow \frac{C+1}{F} = \frac{4}{4} \rightarrow C + 1 = F$$

So in this case, we need to solve this system:

$$\begin{cases} C + 1 = F \\ FC + F = 36 \rightarrow F(C+1) = 36 \rightarrow F^2 = 36 \end{cases} \qquad \begin{cases} F^* = 6 \\ C^* = F - 1 = 5 \end{cases}$$

Income Effect on F: $F_{final\,basket} - F_{decomposition\,basket} = 3 - 6 = -3$
Substitution Effect on F: $F_{decomposition\,basket} - F_{initial\,basket} = 6 - 12 = -6$

Exercise 3

Suppose the market for rental cars has two segments, business travelers and vacation travelers.
The demand curve for rental cars by business travelers is $Q_b = 35 - 0.25P$, where Q_b is the quantity demanded by business travelers (in thousands of cars) when the rental price is P dollars per day. No business customers will rent cars if the price exceeds \$140 per day.

The demand curve for rental cars by vacation travelers is $Q_v = 120 - 1.5P$, where Q_v is the quantity demanded by vacation travelers (in thousands of cars) when the rental price is P dollars per day. No vacation customers will rent cars if the price exceeds $80 per day.

a) Fill in the table to find the quantities demanded in the market at each price.

Price ($/day)	Business (thousands of cars/day)	Vacation (thousands of cars/day)	Market Demand (thousands of cars/day)
100			
90			
80			
70			
60			
50			

b) Graph the demand curves for each segment, and draw the market demand curve for rental cars.

c) Describe the market demand curve algebraically. In other words, show how the quantity demanded in the market Q_m depends on P. Make sure that your algebraic equation for the market demand is consistent with your answers to parts (a) and (b).

d) If the price of a rental car is $60, what is the consumer surplus in each market segment?

Solution

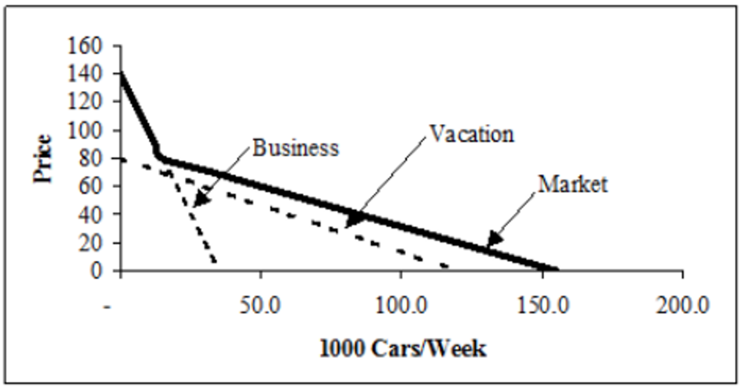
a) In order to fill the table, you can implement the following reasoning:

Price ($/day)	Business (thousands of cars/day)	Vacation (thousands of cars/day)	Market Demand (thousands of cars/day)
P	$Q_b = 35 - 0.25P$	$Q_v = 120 - 1.5P$	$Q_b + Q_v$

The complete version of the table is below:

Price ($/day)	Business (thousands of cars/day)	Vacation (thousands of cars/day)	Market Demand (thousands of cars/day)
100	10.0	-	10.0
90	12.5	-	12.5
80	15.0	-	15.0
70	17.5	15.0	32.5
60	20.0	30.0	50.0
50	22.5	45.0	67.5

b) Graph:



c) For a price greater than \$80, vacation traveler's demand will be zero. So above $P = 80$, market demand is $Q_b = 35 - 0.25P$. For price between \$0 and \$80, market demand is the sum of the vacation and business demand, $Q_m = Q_b + Q_V$, or

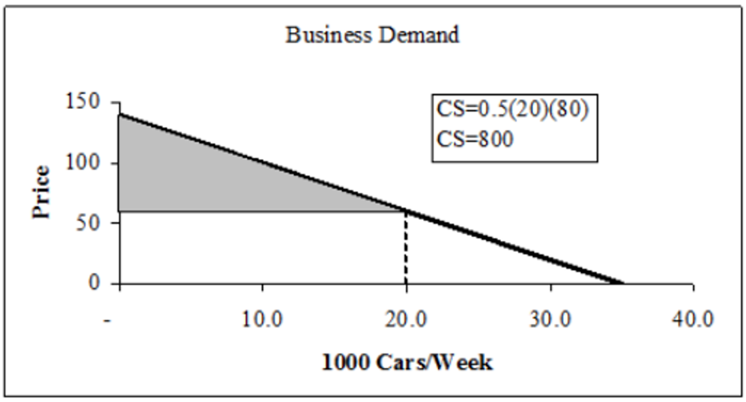
$$Q_m = 35 - 0.25P + 120 - 1.5P$$

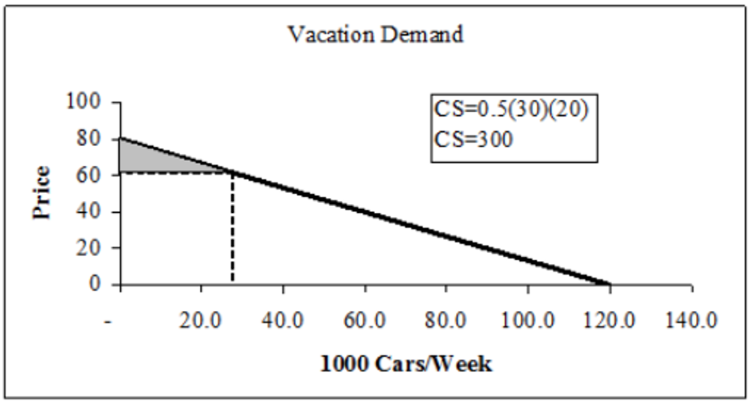
$$Q_m = 155 - 1.75P$$

Above a price of \$140, no purchases will be made so market demand is zero. In summary:

$$Q_m = \begin{cases} 0 & \text{when } P \geq 140 \\ 35 - 0.25P & \text{when } 80 \leq P < 140 \\ 155 - 1.75P & \text{when } P < 80 \end{cases}$$

d) Graphs:





Exercise 4

Terry's utility function over leisure (L) and other goods (Y) is $U(L,Y) = Y + LY$. The associated marginal utilities are $MU_Y = 1 + L$ and $MU_L = Y$. He purchases other goods at a price of \$1, out of the income he earns from working. Show that, no matter what Terry's wage rate, the optimal number of hours of leisure that he consumes is always the same. What is the number of hours he would like to have for leisure?

Solution

If Terry's wage rate is w, then the income he earns from working is $Hw = (24 - L)w$. (Because $T = 24$, hours available in a day, and $T = H + L$, how Terry spend his entire time, between working hours and free hours). Since $P_Y = 1$, the number of unit of other good he purchases is $p_Y Y = Hw$ then, $Y = (24 - L)w$.
Now at an optimal bundle, Terry's $MRS_{L,Y}$, must equal the price ratio $\frac{w}{P_Y} = w$. Therefore the tangency condition tells us that $\frac{Y}{1+L} = w$.

$$\begin{cases} Y = (24 - L)w \\ \frac{Y}{1+L} = w \rightarrow Y = w(1+L) \end{cases}$$

$$w(1+L) = (24 - L)w \rightarrow 1 + L = 24 - L \rightarrow L = \frac{23}{2} = 11.5$$

This means that the optimal amount of leisure is $L = 11.5$. You can see this does not depend on the wage rate.

Exercise 5

Raymond consumes leisure (L hours per day) and other goods (Y units per day), with preferences described by $U(L,Y) = 2\sqrt{L} + Y$. The associated marginal utilities are $MU_Y = 1$ and $MU_L = \frac{1}{\sqrt{L}}$. The price of other goods is 1 euro per unit. The wage rate is w euros per hour.

a) Show how the number of units of leisure Raymond chooses depends on the wage rate.

b) How does Raymond's daily income depend on the wage rate?

c) Does Raymond work more when the wage rate rises?

Solution

a) For this utility function, it turns out that the amount of leisure can be determined from the tangency condition alone. The tangency condition for an optimum is:

$$\frac{MU_L}{MU_Y} = \frac{w}{1} \rightarrow \frac{1}{\sqrt{L}} = w$$

Thus $w^2 = \frac{1}{L}$, or $L = \frac{1}{w^2}$.

b) When Raymond consumes L units of leisure, he works $(24 - L)$ hours, and receives an income of $w(24-L)$ euros per day. His expenditure on other goods is Y euros per day. His budget constraint will have income equal to expenditures, or $w(24-L) = Y$. In (a) we learned from the tangency condition that $L = \frac{1}{w^2}$; substituting this into the budget equation reveals that:

$$\begin{cases} L = \frac{1}{w^2} \\ w(24 - L) = Y \rightarrow w(24 - \frac{1}{w^2}) = Y \rightarrow Y = 24w - \frac{1}{w} \end{cases}$$

c) We can answer this in two ways. First from part (a) we see that Raymond consumes less leisure as the wage rises. Thus he works more as the wage rate rises.

$$L \downarrow = \frac{1}{w^2 \uparrow}$$

Alternatively, Raymond works $(24 - L)$ hours per day, i.e., $24 - \frac{1}{w^2}$ hours per day: this increases as w rises.

$$H \uparrow = 24 - \frac{1}{w^2 \uparrow}$$

Microeconomics
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Exercise 1

At first glance, you might think that when a production function has a diminishing marginal rate of technical substitution of labor for capital, it must also have diminishing marginal products of capital and labor. Show that this is not true, using the production function $Q = LK$.

Solution

First of all, let us compute the marginal products of labor and capital:

$$MP_L = \frac{\partial f(L,K)}{\partial L} = K$$

$$MP_K = \frac{\partial f(L,K)}{\partial K} = L$$

We can notice that both are costant respectively in labor and capital: when L increases, MP_L remains constant; as well as, also when K increases, MP_K remains constant. Now let us compute the marginal rate of technical substitution of labor for capital:

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L}$$

We can notice that it diminishes as L increases and K falls as we move along an isoquant. This exercise demonstrates that it is possible to have a diminishing marginal rate of technical substitution even though both of the marginal products are constant. The distinction is that in analyzing $MRTS_{L,K}$, we move along an isoquant, while in analyzing MP_L and MP_K, total output can change.

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Exercise 2

Does a Cobb-Douglas production function, $Q = AL^{\alpha}K^{\beta}$ exhibit increasing, decreasing, or constant returns to scale?

Solution

Let us remeber that the returns to scale is the concept that tells us the percentage by which output will increase when all inputs are increased by a given percentage. In order to understand which kind of returns to scale a given function exhibit, let us increase all input quantities by the same proportional amount λ:

$$f(L, K) = AL^{\alpha}K^{\beta} \rightarrow f(\lambda L, \lambda K) = A(\lambda L)^{\alpha}(\lambda K)^{\beta}$$

We can now distinguish three cases:

- $A(\lambda L)^{\alpha}(\lambda K)^{\beta} > \lambda(AL^{\alpha}K^{\beta}) \rightarrow \lambda^{\alpha+\beta} > \lambda \rightarrow \alpha + \beta > 1$, the function exhibits an **increasing** return to scale,
- $A(\lambda L)^{\alpha}(\lambda K)^{\beta} = \lambda(AL^{\alpha}K^{\beta}) \rightarrow \lambda^{\alpha+\beta} = \lambda \rightarrow \alpha + \beta = 1$, the function exhibits an **constant** return to scale,
- $A(\lambda L)^{\alpha}(\lambda K)^{\beta} < \lambda(AL^{\alpha}K^{\beta}) \rightarrow \lambda^{\alpha+\beta} < \lambda \rightarrow \alpha + \beta < 1$, the function exhibits an **decreasing** return to scale.

This shows that the sum of the exponents $\alpha+\beta$ in the Cobb-Douglas production function determines whether returns to scale are increasing, constant, or decreasing. For this reason, economists have paid considerable attention to estimating this sum when studying production functions in specific industries.

Exercise 3

Derive the elasticity of substitution for a CobbDouglas production function $f(L, K) = AL^{\alpha}K^{\beta}$.

Solution

The elasticity of a production function is:

$$\sigma = \frac{\%\Delta\frac{K}{L}}{\%\Delta MRTS_{L,K}} = \frac{\frac{\Delta\frac{K}{L}}{\frac{K}{L}}}{\frac{\Delta MRTS_{L,K}}{MRTS_{L,K}}} = \frac{\Delta\frac{K}{L}}{\Delta MRTS_{L,K}}\frac{MRTS_{L,K}}{\frac{K}{L}}$$

For this specific function,

$$MP_L = \alpha AL^{\alpha-1}K^{\beta}$$
$$MP_K = \beta AL^{\alpha}K^{\beta-1}$$

$$MRTS_{L,K} = \frac{\alpha}{\beta}\frac{K}{L}$$

Then,

$$\sigma = \frac{\Delta \frac{K}{L}}{\Delta MRTS_{L,K}} \frac{MRTS_{L,K}}{\frac{K}{L}} = \frac{\Delta \frac{K}{L}}{\frac{\alpha}{\beta}\Delta \frac{K}{L}} \frac{\frac{\alpha}{\beta}\frac{K}{L}}{\frac{K}{L}} = 1$$

We can conclude that a Coob-Douglas production function has an elasticity of substitution equal to 1.

Exercise 4

Suppose the production function for automobiles is $Q = LK$ where Q is the quantity of automobiles produced per year, L is the quantity of labor (man-hours) and K is the quantity of capital (machine-hours).

a) What is the general equation for the isoquant corresponding to any level of output Q?

b) Sketch the isoquant corresponding to a quantity of $Q = 100$?

c) Does the isoquant exhibit diminishing marginal rate of technical substitution?

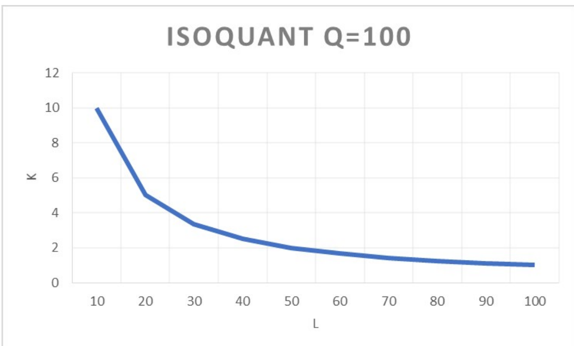
d) Does this production function has a diminishing return to scale?

Solution

a) Let us fix a certain level $\bar{Q}$, the equation for the isoquant corresponding to this level is:

$$\bar{Q} = KL \rightarrow K = \frac{\bar{Q}}{L}$$

b) **Graph**



c) Yes, the isoquant exhibits a diminishing marginal rate of technical substitution:

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{K}{L}$$

We can notice that it diminishes as L increases and K falls as we move along an isoquant.

d) No, the function does not exhibit a diminishing return to scale, but an increasing one:

$$(\lambda K)(\lambda L) > \lambda(KL) \rightarrow \lambda^2 > \lambda$$

Exercise 5

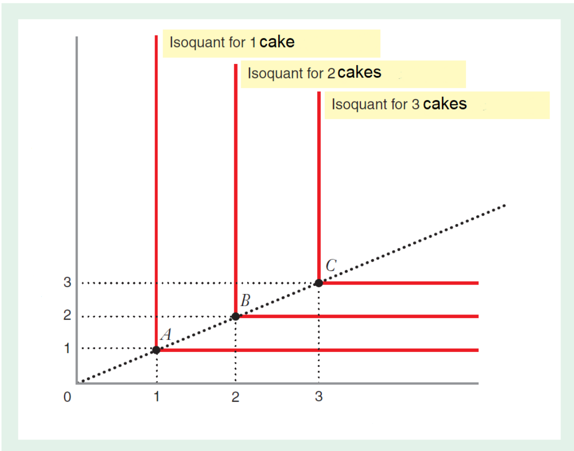
To produce cake, you need eggs E and premixed ingredients I. Every cake needs exactly one egg and one package of ingredients. When you add two eggs to one package of ingredients, you produce only one cake. Similarly, when you have only one egg, you can't produce two cakes even though you have two packages of ingredients.

a) Draw several isoquants of the cake production function.

b) Write a mathematical expression for this production function. What can you say about returns to scale of this function?

Solution

a) **Graph**



b) The mathematical expression of this production function is:

$$Q = min(L, K)$$

This production function ehibits a constant return to scale:

$$min(\lambda L, \lambda K) = \lambda[min(L, K)]$$

Exercise 6

Consider a CES production function given by $Q = (K^{\frac{1}{2}} + L^{\frac{1}{2}})^2$.

a) What is the elasticity of substitution for this production function?

b) Does this production function exhibit increasing, decreasing, or constant returns to scale?

c) Suppose that the production function took the form $Q = (100 + K^{\frac{1}{2}} + L^{\frac{1}{2}})^2$. Does this production function exhibit increasing, decreasing, or constant returns to scale?

Solution

a) The CES production function has a peculiar functional form:

$$Q = (K^\rho + L^\rho)^{\frac{1}{\rho}}$$

It can be consider as a *father* function from which we can derive, with some transformation, all the typical functional form that we use in the study of Microeconomics. Its elasticity of substitution can be easily derive from this formula:

$$\sigma = \frac{1}{1-\rho}$$

In our case, $\rho = \frac{1}{2}$, then:

$$\sigma = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

b) It exhibits a constant return to scale:

$$[(\lambda K)^{\frac{1}{2}} + (\lambda L)^{\frac{1}{2}}]^2 = \lambda(K^{\frac{1}{2}} + L^{\frac{1}{2}})^2$$

$$[\lambda^{\frac{1}{2}}(K^{\frac{1}{2}} + L^{\frac{1}{2}})]^2 = \lambda(K^{\frac{1}{2}} + L^{\frac{1}{2}})^2$$

$$\lambda(K^{\frac{1}{2}} + L^{\frac{1}{2}})^2 = \lambda(K^{\frac{1}{2}} + L^{\frac{1}{2}})^2$$

c) It exhibits an increasing return to scale:

$$[100 + (\lambda K)^{\frac{1}{2}} + (\lambda L)^{\frac{1}{2}}]^2 < \lambda(100 + K^{\frac{1}{2}} + L^{\frac{1}{2}})^2$$

$$[100 + \lambda^{\frac{1}{2}}(K^{\frac{1}{2}} + L^{\frac{1}{2}})]^2 < \lambda(100 + K^{\frac{1}{2}} + L^{\frac{1}{2}})^2$$

Microeconomics
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Exercise 1

Consider the production function $Q = 50\sqrt{LK}$. What is the short-run total cost curve for this production function when capital is fixed at a level $\bar{K}$, and the input prices of labor and capital are $w = 25$ and $r = 100$, respectively?

Solution

First of all we have to compute the cost-minimizing quantity of labour in the short run, the optimal amount of labour, in the following way:

$$Q = 50\sqrt{L\bar{K}} \rightarrow L = \frac{Q^2}{2500\bar{K}}$$

We can obtain the short-run total cost curve directly from:

$$STC(Q) = wL + r\bar{K} = 25L + 100\bar{K} = 25\frac{Q^2}{2500\bar{K}} + 100\bar{K} = \frac{Q^2}{100\bar{K}} + 100\bar{K}$$

The total variable and total fixed cost curves follow: $TVC(Q) = \frac{Q^2}{100\bar{K}}$ and $TFC = 100\bar{K}$. Note that, holding Q constant, total variable cost is decreasing in the quantity of capital $\bar{K}$. The reason is that, for a given amount of output, a firm that uses more capital can reduce the amount of labor it employs. Since TVC is the firm's labor expense, it follows that TVC should decrease in $\bar{K}$.

Exercise 2

The text discussed the expansion path as a graph that shows the cost-minimizing input quantities as output changes, holding fixed the prices of inputs. What the text didn't say

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is that there is a *different* expansion path for each pair of input prices the firm might face. In other words, how the inputs vary with output depends, in part, on the input prices. Consider, now, the expansion paths associated with two distinct pairs of input prices, (w_1, r_1) and (w_2, r_2).
Assume that at both pairs of input prices, we have an interior solution to the cost-minimization problem for any positive level of output. Also assume that the firms isoquants have no kinks in them and that they exhibit diminishing marginal rate of technical substitution. Could these expansion paths ever cross each other at a point other than the origin $(L = 0, K = 0)$?

Solution

No, if the MRTS is diminishing, the expansion path for different input price combinations cannot cross. To understand why, imagine for the moment that they did cross at some point. Recall that the expansion path traces out the cost-minimizing combinations of inputs as output increases. Essentially the expansion path traces out all of the tangencies between the isocost lines and isoquants. These tangencies occur at the point where

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

If the expansion paths cross at some point, then the costminimizing combination of inputs must be identical with both sets of prices. This would require that

$$\frac{MP_L}{w_1} = \frac{MP_K}{r_1} \text{ and } \frac{MP_L}{w_2} = \frac{MP_K}{r_2}$$

Unless these pairs of prices are proportional, it is not possible for both of these equations to hold. Therefore, it is not possible for the expansion paths to cross unless the prices are proportional, in which case the two expansion paths will be identical.

Exercise 3

The processing of payroll for the $10,000$ workers in a large firm can either be done using 1 hour of computer time (denoted by K) and no clerks or with 10 hours of clerical time (denoted by L) and no computer time. Computers and clerks are perfect substitutes; for example, the firm could also process its payroll using $\frac{1}{2}$ hour of computer time and 5 hours of clerical time.

a) Sketch the isoquant that shows all combinations of clerical time and computer time that allows the firm to process the payroll for $10,000$ workers.

b) Suppose computer time costs \$5 per hour and clerical time costs \$7.50 per hour. What are the cost-minimizing choices of L and K? What is the minimized total cost of processing the payroll?

c) Suppose the price of clerical time remains at \$7.50 per hour. How high would the price of an hour of computer time have to be before the firm would find it worthwhile to use only clerks to process the payroll?

Solution

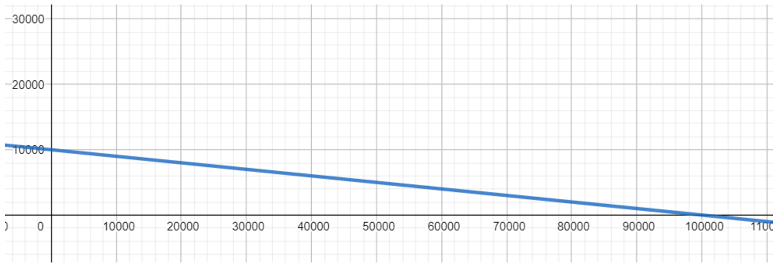
a) The production function for the processing of payroll, and the general function of an isoquant, are equal to:

$$Q(L,K) = K + \frac{1}{10}L \rightarrow K = \bar{Q} - \frac{1}{10}L$$

If we fix the level $\bar{Q} = 10,000$, we get:

$$K = 10,000 - \frac{1}{10}L$$

Which is represented in the following graph:



b) We know that for an optimal interior solution this is the condition to satisfy:

$$\text{slope of isoquant} = \text{slope of isocost line}$$

$$-MRTS_{L,K} = -\frac{w}{r}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\text{ratio of marginal product} = \text{ratio of input prices}$$

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

$$MP_L \text{ relative to its price} = MP_K \text{ relative to its price}$$

We can then compute the marginal productivities, $MP_L = \frac{1}{10}$ and $MP_K = 1$ and verify the equality:

$$\frac{1}{(10)(7.50)} < \frac{1}{5} \rightarrow \frac{MP_L}{w} < \frac{MP_K}{r} \rightarrow \text{Corner Solution } (0, K^*)$$

In order to prepare payrolls, only computers will be used.

c) We have to find an r such that the firm would find it worthwhile to use only clerks to process the payroll, namely such that the following disequality would hold:

$$\frac{MP_L}{w} > \frac{MP_K}{r} \rightarrow \text{Corner Solution } (L^*, 0)$$

$$\frac{1}{(10)(7.50)} > \frac{1}{r} \rightarrow r^* > 75$$

The firm would find it convenient to use only clerks to process the payroll if the price of an hour of computer time will be higher than \$75.

Exercise 4

A firm has the production function $Q = LK$. For this production function, $MP_L = K$ and $MP_K = L$. The firm initially faces input prices $w = \$1$ and $r = \$1$ and is required to produce $Q = 100$ units. Later the price of labor w goes up to \$4. Find the optimal input combinations for each set of prices and use these to calculate the firm's price elasticity of demand for labor over this range of prices.

Solution

In order to find the optimal input combinations, we have to impose the tangency condition for the interior solution, and a specific isoquant. In our case, we know

$$\begin{cases} MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r} \rightarrow \text{Tangency Condition} \\ \bar{Q} = f(L,K) \rightarrow \text{Level of Production} \end{cases}$$

We can now solve the two systems with the information that we have:

1. $w_1 = 1$

$$\begin{cases} \frac{MP_L}{MP_K} = \frac{w_1}{r} \\ \bar{Q} = KL \end{cases} \quad \begin{cases} \frac{K}{L} = \frac{1}{1} \rightarrow K = L \\ 100 = KL \rightarrow K^2 = 100 \end{cases}$$

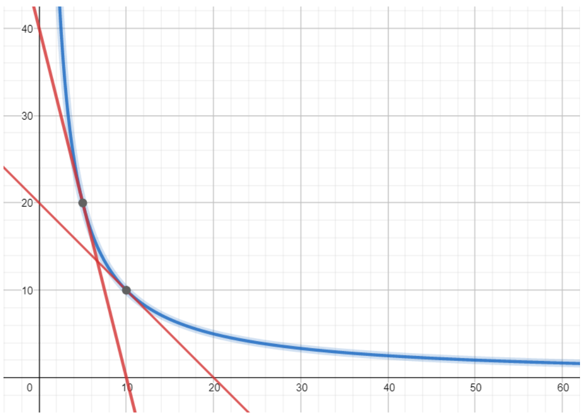
$$\begin{cases} L_1^* = 10 \\ K_1^* = 10 \end{cases}$$

2. $w_2 = 4$

$$\begin{cases} \frac{MP_L}{MP_K} = \frac{w_2}{r} \\ \bar{Q} = KL \end{cases} \quad \begin{cases} \frac{K}{L} = \frac{4}{1} \rightarrow K = 4L \\ 100 = KL \rightarrow (4L)(L) = 100 \rightarrow L^2 = \frac{100}{4} = 25 \end{cases}$$

$$\begin{cases} L_2^* = 5 \\ K_2^* = 20 \end{cases}$$

In the next graph it is possible to observe the optimal input combinations for each set of prices:



Let us now move to the calculation of the firm's price elasticity of demand for labor over this range of prices:

$$\varepsilon_{L,w} = \frac{\Delta L}{\Delta w}\frac{w}{L} = \frac{L_2 - L_1}{w_2 - w_1}\frac{w}{L}$$

In this specific case,

$$\varepsilon_{L_1,w_1} = \frac{5-10}{4-1}\frac{1}{10} = -\frac{5}{3}\frac{1}{10} = -\frac{1}{6}$$

$$\varepsilon_{L_2,w_2} = \frac{5-10}{4-1}\frac{4}{5} = -\frac{5}{3}\frac{4}{5} = -\frac{4}{3}$$

Exercise 5

Suppose the input demand curves of a firm are:

$$L = \frac{Q}{30}\sqrt{\frac{r}{w}}$$

$$K = \frac{Q}{30}\sqrt{\frac{w}{r}}$$

Derive the production function of such firm.

Solution

In order to derive the production function, let us rewrite the labour demand curve in the following way:

$$L = \frac{Q}{30}\sqrt{\frac{r}{w}} \rightarrow Q = 30L\sqrt{\frac{w}{r}}$$

Now, let us rewrite the capital demand curve, expliciting the square root of the ratio between the input prices:

$$K = \frac{Q}{30}\sqrt{\frac{w}{r}} \rightarrow \sqrt{\frac{w}{r}} = 30\frac{K}{Q}$$

Doing so, we now can substitute the the square root of the ratio between the input prices into the first function, in order to have the production function:

$$Q = 30L30\frac{K}{Q}$$

$$Q^2 = 900LK$$

$$Q = 30\sqrt{L}\sqrt{K}$$

Microeconomics
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Exercise 1

Suppose that a firm has a short-run total cost curve given by $STC = 100 + 20Q + Q^2$, where the total fixed cost is 100 and the total variable cost is $20Q+Q^2$. The corresponding short-run marginal cost curve is $SMC = 20 + 2Q$. All of the fixed cost is sunk.

a) What is the equation for average variable cost (AVC)?

b) What is the minimum level of average variable cost?

c) What is the firm's short-run supply curve?

Solution

a) Average variable cost is total variable cost divided by output. Thus,

$$AVC(Q) = \frac{SVC(Q)}{Q} = \frac{20Q + Q^2}{Q} = 20 + Q$$

b) We know that the minimum level of average variable cost occurs at the point at which AVC and SMC are equal in this case,

$$AVC(Q) = SMC(Q) \rightarrow 20 + Q = 20 + 2Q \rightarrow Q_{min} = 0$$
$$AVC(Q_{min}) = 20$$

c) For prices below 20 (the minimum level of average variable cost), the firm will not produce. For prices above 20, we can find the supply curve by equating price to marginal cost and solving for Q:

$$P = SMC(Q) \rightarrow P = 20 + 2Q \rightarrow Q = \frac{1}{2}P - 10$$

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The firm's short-run supply curve, which we denote by $s(P)$, is thus:

$$s(P) = \begin{cases} 0 & P < 20 \\ \frac{1}{2}P - 10 & P \geq 20 \end{cases}$$

Exercise 2

Consider the previous exercise and

a) Suppose that $SFC = 36$, while $NSFC = 64$. What is the firm's average nonsunk cost curve?

b) What is the minimum level of average nonsunk cost?

c) What is the firm's short-run supply curve?

Solution

a) The average nonsunk cost curve is

$$ANSC = AVC + \frac{NSFC}{Q} = 20 + Q + \frac{64}{Q}$$

b) The average nonsunk cost curve $ANSC$ reaches its minimum when average nonsunk cost equals short-run marginal cost:

$$ANSC = SMC \rightarrow 20+Q+\frac{64}{Q} = 20+2Q \rightarrow \frac{64}{Q} = 2Q-Q \rightarrow Q^2 = 64 \rightarrow Q_{min} = 8$$

$$ANSC(Q_{min}) = 20 + 8 + \frac{64}{8} = 36$$

c) For prices below the minimum level of ANSC (i.e., for $P = 36$), the firm does not produce. For prices above this level, the firm's profit-maximizing quantity is given by equating price to marginal cost and solving for Q:

$$P = SMC(Q) \rightarrow P = 20 + 2Q \rightarrow Q = \frac{1}{2}P - 10$$

The firm's short-run supply curve, which we denote by $s(P)$, is thus:

$$s(P) = \begin{cases} 0 & P < 36 \\ \frac{1}{2}P - 10 & P \geq 36 \end{cases}$$

When the market price is between 36 and 40, the firm will continue to produce in the short run, even though its economic profit is negative. Its losses from operating will be less than its losses if it shuts down.

Exercise 3

A market consists of 300 identical firms, and the market demand curve is given by $D(P) = 60 - P$. Each firm has a short-run total cost curve $STC = 0.1 + 150Q^2$, and all fixed costs are sunk. The corresponding short-run marginal cost curve is $SMS = 300Q$, and the corresponding average variable cost curve is $AVC = 150Q$. The minimum level of AVC is 0; thus, a firm will continue to produce as long as price is positive. (You can verify this by sketching the SMC and AVC curves.)
What is the short-run equilibrium price in this market?

Solution

Each firm's profit-maximizing quantity is given by equating marginal cost and price:

$$P = SMC(Q) \rightarrow P = 300Q \rightarrow Q = \frac{P}{300}$$

Thus the supply curve $s(P)$ of an individual firm is

$$s(P) = \frac{P}{300}$$

Since the firms in this market are all identical, short-run market supply $S(P)$ equals

$$S(P) = ns(P) = 300\frac{P}{300} = P$$

The short-run equilibrium occurs where market supply equals market demand, thus

$$S(P) = D(P) \rightarrow P = 60 - P \rightarrow 2P = 60 \rightarrow P = 30$$

Exercise 4

In a market, all firms and potential entrants are identical. Each has a long-run average cost curve $AC(Q) = 40 - Q + 0.01Q^2$ and a corresponding long-run marginal cost curve $MC(Q) = 40 - 2Q + 0.03Q^2$, where Q is thousands of units per year. The market demand curve is $D(P) = 25,000 - 1,000P$ where $D(P)$ is also measured in thousands of units. Find the long-run equilibrium quantity per firm, price, and number of firms.

Solution

The long-run competitive equilibrium satisfies the following three equations:

$$\begin{cases} P = MC(Q) \rightarrow \text{ profit maximization} \\ P = AC(Q) \rightarrow \text{ zero profit} \\ S(P) = D(P) \rightarrow \text{ supply equals demand (market clearing)} \end{cases}$$

$$\begin{cases} P = 40 - 2Q + 0.03Q^2 \\ P = 40 - Q + 0.01Q^2 \\ nQ = 25,000 - 1,000P \rightarrow n = \frac{25,000 - 1,000P}{Q} \end{cases}$$

$$40 - 2Q + 0.03Q^2 = 40 - Q + 0.01Q^2$$
$$(0.03 - 0.01)Q^2 = (2 - 1)Q$$
$$Q^* = \frac{1}{0.02} = 50$$
$$P = 40 - 50 + 0.01(2500) = 40 - 50 + 25 \rightarrow P^* = 15$$
$$n = \frac{25,000 - 1,000(15)}{50} = \frac{25,000 - 15,000}{50} = \frac{10,000}{50} \rightarrow n^* = 200$$

Exercise 5

A producer operating in a perfectly competitive market has chosen his output level to maximize profit. At that output, his revenue and costs are as follows:

- Revenue \$200
- Variable costs \$120
- Sunk fixed costs \$60
- Nonsunk fixed costs \$40

Calculate his producer surplus and his profits.
Extra Exercise: Which (if either) of these should he use to determine whether he should exit the market in the short run? Briefly explain.

Solution

In order to compute producer surplus, it is enough to compute the difference between the firm's total revenue and its total nonsunk cost:

$$PS = TR - (VC + NSFC) = 200 - (120 + 40) = 40\$$$

While, to compute producer profit, we simply have to subtract total costs to total revenues:

$$\pi = TR - TC = 200 - (120 + 60 + 40) = -20\$$$

Extra Exercise: To decide whether to operate or shut down, the firm should look at producer surplus (rather than profit). Producer surplus (40) shows how much better off he would be operating (with a profit = -20) than shutting down (with a profit = -60). So he should stay in business in the short run; he will lose money, but not as much as if her were to shut down.

Microeconomics
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Exercise 1

A monopolist faces a demand curve $P = 210 - 4Q$ and initially faces a constant marginal cost $MC = 10$.

a) Calculate the profit-maximizing monopoly quantity and compute the monopolist's total revenue at the optimal price.

b) Suppose that the monopolist's marginal cost increases to $MC = 20$. Verify that the monopolist's total revenue goes down.

c) Suppose that all firms in a perfectly competitive equilibrium had a constant marginal cost $MC = 10$. Find the long-run perfectly competitive industry price and quantity.

d) Suppose that al firms' marginal costs increased to $MC = 20$. Verify that the increase in marginal cost causes total industry revenue to go up.

Solution

a) With demand $P = 210 - 4Q$, $MR = 210 - 8Q$. Setting $MR = MC$ implies

$$210 - 8Q = 10 \rightarrow Q = 25$$

$$P = 210 - 4(25) \rightarrow P = 110$$
$$TR = 110(25) = 2750$$

b) If $MC = 20$, than setting $MR = MC$ implies

$$210 - 8Q = 20 \rightarrow Q = 23.75$$

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$$P = 210 - 4(20) \rightarrow P = 115$$
$$TR = 115(23.75) = 2731,25$$

Therefore, the increase in marginal cost will result in lower total revenue for the firm.

c) Competitive firms produce until $P = MC$, so in this case we know the market price would be

$$P_1 = 10$$
$$210 - 4Q = 10 \rightarrow Q_1 = 50$$

d) In this case, the market price will be

$$P_2 = 20$$
$$210 - 4Q = 20 \rightarrow Q_2 = 47,50$$

Let us now compare total revenue first and after the increase in marginal cost:

$$TR_1 = 10(50) = 500$$
$$TR_2 = 20(47,50) = 950$$
$$500 < 950$$
$$TR_1 < TR_2$$

Total industry revenue increases in the perfectly competitive market after the increases in the perfectly competitive market after the increase in marginal cost.

Exercise 2

Let a monopolist face a linear demand, $P_0(Q) = 100-2Q$ and marginal cost $MC(Q) = 20$. Suppose that now demand shifts to $P_1(Q) = 120 - 2Q$. How does the equilibrium price change?

Solution

For inverse demand curve $P_0(Q)$, we have

$$MR_0(Q) = 100 - 4Q$$

Setting this equal to marginal cost, we have

$$100 - 4Q = 20 \rightarrow Q_0 = 20$$
$$P = 100 - 2(20) \rightarrow P_0 = 60$$

For inverse demand curve $P_1(Q)$, we have

$$MR_1(Q) = 120 - 4Q$$

Setting this equal to marginal cost, we have

$$120 - 4Q = 20 \rightarrow Q_1 = 25$$
$$P = 100 - 2(25) \rightarrow P_1 = 70$$

Therefore, $P_1 > P_0$ and so the equilibrium price has risen.

Exercise 3

Suppose a monopolist has a constant marginal cost $MC = \$50$ and faces a deman curve $P = 100 - \frac{1}{2}Q$ (which can be rewritten as $Q = 200 - 2P$).

a) Find the profit-maximizing price and quantity for the monopolist using the monopoly midpoint rule.

b) Find the profit-maximizing price and quantity for the monopolist by equating MR to MC.

Solution

a) The monopoly midpoint rule says that we need to compare the vertical intercept of demand to the intercept of marginal cost. Here, we have a vertical intercept of 100 for demand from the expression of inverse demand, $P = 100 - \frac{1}{2}Q$ and we have a vertical intercept of 50 for marginal cost, as it constant at 50. Hence, we have price equal to the midpoint between 50 and 100:

$$P = \frac{50 + 100}{2} \rightarrow P = 75$$

Substituting this into the inverse demand:

$$Q = 200 - 2(75) \rightarrow Q = 50$$

b) From the inverse demand, we can see that the marginal revenue:

$$MR = 100 - Q$$

Equating this to marginal cost we have:

$$100 - Q = 50 \rightarrow Q = 50$$

Substituting into the expression for the inverse demand we have

$$P = 100 - \frac{50}{2} \rightarrow P = 75$$

This answer squares with that of the monopoly midpoint rule in (a).

Exercise 4

A homogeneous products duopoly faces a market demand function given by $P = 300 - 3Q$, where $Q = Q_l + Q_2$. Both firms have a constant marginal cost $MC = 100$.

a) What is Firm l's profit-maximizing quantity, given that Firm 2 produces an output of 50 units per year? What is Firm 1's profit-maximizing quantity when Firm 2 produces 20 units per year?

b) Derirve the equation of each firm's reaction curve and then graph these curves.

Solution

a) Demand is given by $P = 300 - 3Q_1 - 3Q_2$. If $Q_2 = 50$, then

$$P = 300 - 3Q_1 - 3(50) \rightarrow P = 150 - 3Q_1$$

Setting $MR = MC$ implies

$$150 - 6Q_1 = 100 \rightarrow Q_1 = 8.33$$

Instead, if $Q_2 = 20$

$$P = 300 - 3Q_1 - 3(20) \rightarrow P = 240 - 3Q_1$$

Setting $MR = MC$ implies

$$240 - 6Q_1 = 100 \rightarrow Q_1 = 23.33$$

b) For Firm 1, we know:

$$P = (300 - 3Q_2) - 3Q_1$$

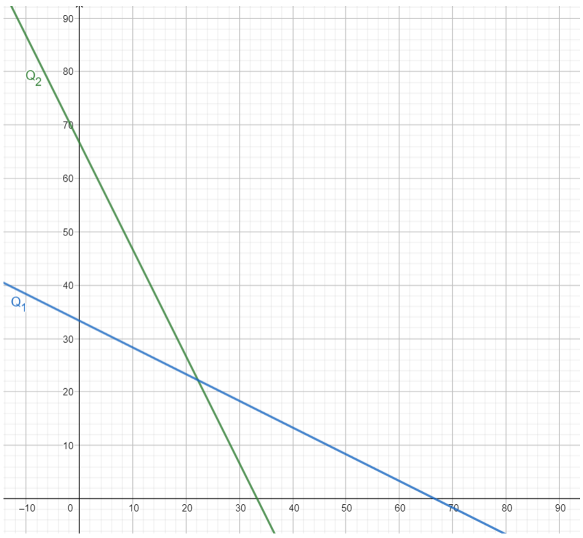
Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100 \rightarrow 6Q_1 = 200 - 3Q_2$$

$$Q_1^{BR} = 33.33 - \frac{1}{2}Q_2$$

Since the marginal costs are the same for both firms, symmery implies:

$$Q_2^{BR} = 33.33 - \frac{1}{2}Q_1$$



Microeconomics
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May 11, 2021

Exercise 1

A homogeneous products duopoly faces a market demand function given by $P = 300-3Q$, where $Q = Q_l + Q_2$. Both firms have a constant marginal cost $MC = 100$.

a) What is Firm l's profit-maximizing quantity, given that Firm 2 produces an output of 50 units per year? What is Firm 1's profit-maximizing quantity when Firm 2 produces 20 units per year?

b) Derirve the equation of each firm's reaction curve and then graph these curves.

c) What is the Cournot equilibrum, quantity per firm and price in this market?

d) What would the equilibrium price in this market be if it were perfectly competitive?

e) What would the equilillrium price in this market be if the two firms colluded to set the monopoly price?

f) What is the Bertrand equilibrium price is this market?

g) What are the Cournot equilibrium quantities and industry price when one firm has a marginal cost of 100 but the other firm has a marginal cost of 90?

Solution

a) Demand is given by $P = 300 - 3Q_1 - 3Q_2$. If $Q_2 = 50$, then

$$P = 300 - 3Q_1 - 3(50) \rightarrow P = 150 - 3Q_1$$

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Setting $MR = MC$ implies

$$150 - 6Q_1 = 100 \rightarrow Q_1 = 8.33$$

Instead, if $Q_2 = 20$

$$P = 300 - 3Q_1 - 3(20) \rightarrow P = 240 - 3Q_1$$

Setting $MR = MC$ implies

$$240 - 6Q_1 = 100 \rightarrow Q_1 = 23.33$$

b) For Firm 1, we know:

$$P = (300 - 3Q_2) - 3Q_1$$

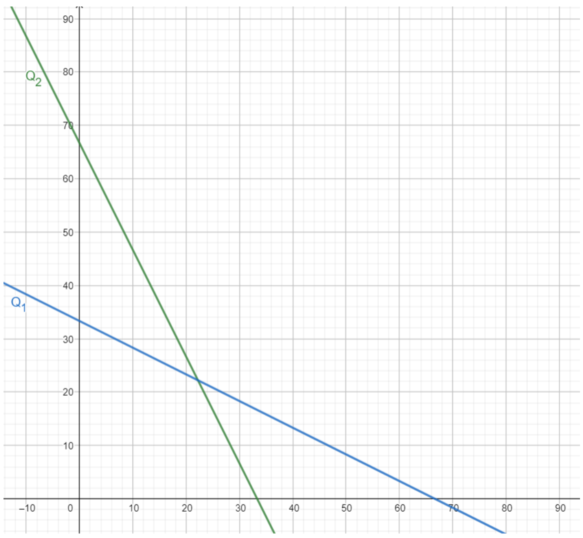
Setting $MR = MC$ implies

$$(300 - 3Q_2) - 6Q_1 = 100 \rightarrow 6Q_1 = 200 - 3Q_2$$

$$Q_1^{BR} = 33.33 - \frac{1}{2}Q_2$$

Since the marginal costs are the same for both firms, symmery implies:

$$Q_2^{BR} = 33.33 - \frac{1}{2}Q_1$$



c) Because of symmetry, in equilibrium both firms will choose the same level of output. Thus we can set $Q_1 = Q_2$ and solve

$$Q_1 = 33.33 - \frac{1}{2}Q_1 \rightarrow \frac{3}{2}Q_1 = 33.33\frac{2}{3}$$

$$Q_1^C = 22.22$$

$$Q_2^C = 22.22$$

Since both firms will choose the same level of output, both firms will produce 22.22 units. Price can be found by substituting the quantity for each firm into market demand. This implies price will be

$$P = 300 - 3(44.44) \rightarrow P^C = 166.67$$

d) If this market were perfectly competitive, then equilibrium would occur at the point where

$$P = MC \rightarrow P^{PC} = 100$$

e) If the firms colluded to set the monopoly price, then

$$300 - 6Q = 100$$

$$Q = \frac{200}{6} = 33.33$$

$$P = 300 - 3(\frac{200}{6}) \rightarrow P^{Collusion} = 200$$

f) If the firms acted as Bertrand oligopolists, the equilibrium would coincide with the perfectly competitive equilibrium of

$$P^{PC} = P^{B} \rightarrow P^{B} = 100$$

g) Suppose $MC_1 = 100$ and $MC_2 = 90$. For Firm 1,

$$(300 - 3Q_2) - 6Q_1 = 100 \rightarrow 6Q_1 = 200 - 3Q_2$$

$$Q_1^{BR} = 33.33 - \frac{1}{2}Q_2$$

For Firm 2,

$$(300 - 3Q_2) - 6Q_2 = 90 \rightarrow 6Q_2 = 210 - 3Q_1$$

$$Q_2^{BR} = 35 - \frac{1}{2}Q_1$$

Solving these two reaction functions simultaneously yields

$$\begin{cases} Q_1^{BR} = 33.33 - \frac{1}{2}Q_2 \\ Q_2^{BR} = 35 - \frac{1}{2}Q_1 \end{cases}$$

$$Q_2 = 35 - \frac{1}{2}(33.33 - \frac{1}{2}Q_2) \rightarrow \frac{3}{4}Q_2 = 35 - 16,67$$

$$Q_2^{C} = 24,44$$

$$Q_1 = 33.33 - \frac{1}{2}(24,44)$$

$$Q_1^{C} = 21,11$$

With these quantities, market price will be

$$P = 300 - 3(21,11 + 24,44) \rightarrow P^{C} = 163,36$$

Exercise 2

The market demand curve in a commodity chemical industry is given by $Q = 600 - 3P$, where Q is the quantity demanded per month and P is the market price in dollars. Firms in this industry supply quantities every month, and the resulting market price occurs at the point at which the quantity demanded equals the total quantity supplied. Suppose there are two firms in this industry, Firm 1 and Firm 2. Each firm has an identical constant marginal cost of \$80 per unit.

a) Find the Cournot equilibrium quantities for each firm. What is the Cournot equilibrium market price?

b) Assuming that Firm 1 is the Stackelberg leader, find the Stackelberg equilibrium quantities for each firm. What is the Stackelberg equilibrium price?

c) Calculate and compare the profit of each firm under the Cournot and Stackelberg equilibria. Under which equilibrium is overall industry profit the greatest, and why?

Solution

a) Let us invert the market demand curve

$$Q = 600 - 3P \rightarrow P = 200 - \frac{1}{3}Q \rightarrow P = 200 - \frac{1}{3}(Q_1 + Q_2)$$

For Firm 1 setting $MR = MC$

$$(200 - \frac{1}{3}Q_2) - \frac{2}{3}Q_1 = 80 \rightarrow 120 - \frac{1}{3}Q_2 = \frac{2}{3}Q_1$$

$$Q_1^{BR} = 180 - \frac{1}{2}Q_2$$

Because of symmetry, in equilibrium both firms will choose the same level of output.

$$Q_2^{BR} = 180 - \frac{1}{2}Q_1$$

Solving these two reaction functions simultaneously yields

$$\begin{cases} Q_1^{BR} = 180 - \frac{1}{2}Q_2 \\ Q_2^{BR} = 180 - \frac{1}{2}Q_1 \end{cases}$$

$$Q_1^{BR} = 180 - \frac{1}{2}(180 - \frac{1}{2}Q_1) \rightarrow \frac{3}{4}Q_1 = 90$$

$$Q_1^C = Q_2^C = 120$$

$$P = 200 - \frac{1}{3}(240) \rightarrow P^C = 120$$

b) Let us start from Firm 2 reaction function, using MR_2

$$(200 - \frac{1}{3}Q_1) - \frac{2}{3}Q_2 = 80 \rightarrow 120 - \frac{1}{3}Q_1 = \frac{2}{3}Q_2$$

$$Q_2^{BR} = 180 - \frac{1}{2}Q_1$$

Now we can compute MR_1:

$$P = 200 - \frac{1}{3}(Q_1 + 180 - \frac{1}{2}Q_1) \rightarrow P = 200 - \frac{1}{6}Q_1$$

$$MR_1 = 140 - \frac{1}{3}Q_1$$

$$140 - \frac{1}{3}Q_1 = 80 \rightarrow Q_1 = (140 - 80)3$$

$$Q^S_{Leader} = 180$$

$$Q_2 = 180 - \frac{1}{2}180 \rightarrow Q^S_{Follower} = 90$$

$$P = 200 - \frac{1}{3}(180 + 90) \rightarrow P^S = 110$$

Notice that this is lower than the Cournot equilibrium price:

$$P^S < P^C$$

c) Let us now compute and compare profits:

$$\pi_i^C = (P - MC)Q_i = (120 - 80)(120) = \$4,800$$

$$\pi^S_{Leader} = (P - MC)Q_1 = (110 - 80)(180) = \$5,400$$

$$\pi^S_{Follower} = (P - MC)Q_2 = (110 - 80)(90) = \$2,700$$

$$\pi^S_{Follower} < \pi_i^C < \pi^S_{Leader}$$

$$\pi^S_{Industry} < \pi^C_{Industry}$$