Consumer Preferences and the Concept of Utility

Adapted from Chapter 3 of Besanko's Microeconomics

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Overview

Motivation

Why study consumer choice?

- Study of how consumers with limited resources choose goods and services
- Helps derive the demand curve for any good or service
- Businesses care about consumer demand curves
- Government can use this to determine how to help and whom to help buy certain goods and services

Consumer Preferences

Consumer Preferences tell us how the consumer would rank (that is, compare the desirability of) any two combinations or allotments of goods, assuming these allotments were available to the consumer at no cost.

These allotments of goods are referred to as **baskets** or **bundles**. These baskets are assumed to be available for consumption at a particular time, place and under particular physical circumstances.

Assumptions

Complete and Transitive

Preferences are **complete** if the consumer can rank any two baskets of goods (A preferred to B; B preferred to A; or indifferent between A and B).

Preferences are **transitive** if a consumer who prefers basket A to basket B, and basket B to basket C also prefers basket A to basket C.

$$A \succ B; B \succ C \Rightarrow A \succ C$$

Assumptions

Monotonicity / Free Disposal

Preferences are **monotonic** if a basket with more of at least one good and no less of any good is preferred to the original basket.

Types of Ranking

Example

Students take an exam. After the exam, the students are ranked according to their performance. An ordinal ranking lists the students in order of their performance (i.e., Harry did best, Joe did second best, Betty did third best, and so on). A cardinal ranking gives the mark of the exam, based on an absolute marking standard (i.e., Harry got 80, Joe got 75, Betty got 74 and so on). Alternatively, if the exam were graded on a curve, the marks would be an ordinal ranking.

The Utility Function

The three assumptions about preferences allow us to represent preferences with a **utility function**.

Utility function

- a function that measures the level of satisfaction a consumer receives from any basket of goods and services.
- assigns a number to each basket so that more preferred baskets get a higher number than less preferred baskets.
- $U = u(y)$

Implications

- An ordinal concept: the precise magnitude of the number that the function assigns has no significance.
- Utility not comparable across individuals.
- Any transformation of a utility function that preserves the original ranking of bundles is an equally good representation of preferences. e.g. $U(y) = \sqrt{y}$ vs $U(y) = \sqrt{y} + 2$ represent the same preferences.

Marginal Utility

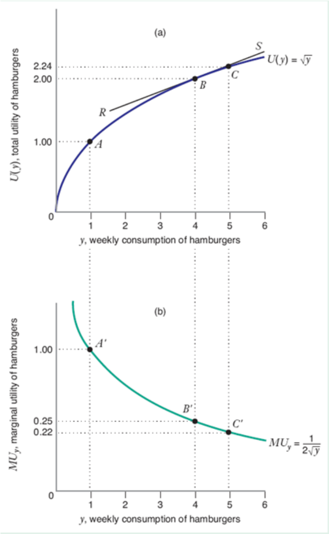
Marginal Utility of a good y

- additional utility that the consumer gets from consuming a little more of y i.e. the rate at which total utility changes as the level of consumption of good y rises
- $MU_y = \frac{\Delta U}{\Delta y}$
- slope of the utility function with respect to y

Diminishing Marginal Utility

The principle of diminishing marginal utility states that the marginal utility falls as the consumer consumes more of a good.

Diminishing Marginal Utility



Marginal Utility

The marginal utility of a good, x, is the additional utility that the consumer gets from consuming a little more of x when the consumption of all the other goods in the consumer's basket remain constant.

$$U(x,y) = x + y$$

$$\frac{\Delta U}{\Delta x}(y \text{ held costant}) = MU_x$$

$$\frac{\Delta U}{\Delta y}(x \text{ held costant}) = MU_y$$

Marginal Utility - Example 1

Example

$$U(H) = 10H – H^2$$

$$MU_H = 10 – 2H$$

Marginal Utility - Example 1

Example

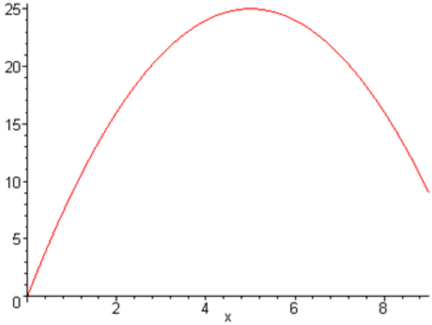
$$U(H) = 10H – H^2$$

$$MU_H = 10 – 2H$$

H	H^2	$U(H)$	MU_H
2	4	16	6
4	16	24	2
6	36	24	-2
8	64	16	-6
10	100	0	-10

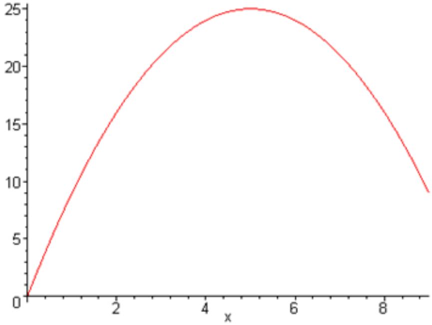
Marginal Utility - Example 2

$$U(H) = 10H - H^2$$

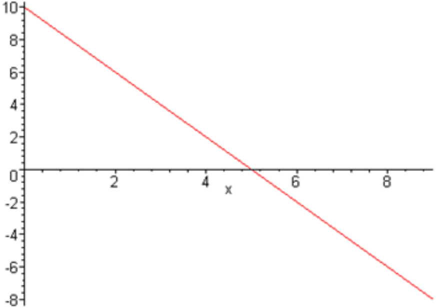


Marginal Utility - Example 2

$$U(H) = 10H - H^2$$



$$MU_H = 10–2H$$



Marginal Utility - Example 3

Example

- The point at which he should stop consuming hotdogs is the point at which $MU_H = 0$
- This gives $H = 5$.
- That is the point where Total Utility is flat.
- You can see that the utility is diminishing.

Marginal Utility - Multiple Goods

$$U = xy^2$$

$$MU_x = y^2$$

$$MU_y = 2xy$$

- More is better?

Marginal Utility - Multiple Goods

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- Diminishing marginal utility?

Marginal Utility - Multiple Goods

$$U = xy^2$$

$$MU_x = y^2$$

$$MU_y = 2xy$$

- More is better? More y more and more x indicates more U so yes it is monotonic
- Diminishing marginal utility?
  - MU of x is not dependent of x. So the marginal utility of x (movies) does not decrease as the number of movies increases.
  - MU of y increases with increase in number of operas (y) so neither exhibits diminishing returns.

Indifference Curves

Indifference Curve

An **Indifference Curve** or **Indifference Set** is the set of all baskets for which the consumer is indifferent.

Indifference Map

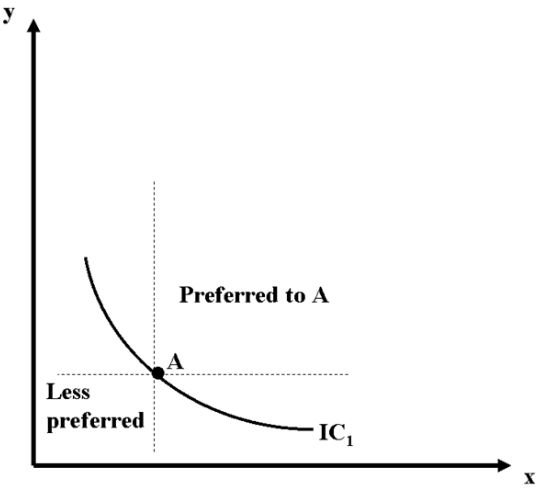
An **Indifference Map** illustrates a set of indifference curves for a consumer.

Indifference Curves

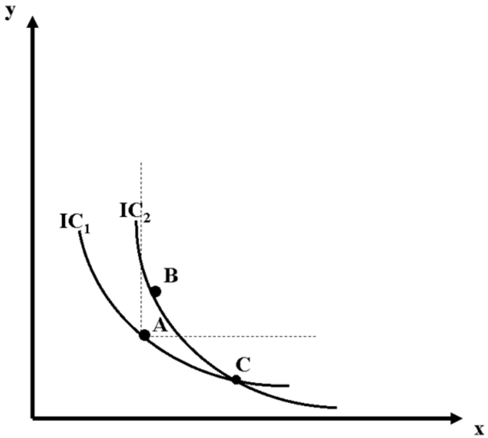
Key Properties

1. **Monotonicity** $\Rightarrow$ indifference curves have negative slope – and indifference curves are not *thick*.
2. **Transitivity** $\Rightarrow$ indifference curves do not cross.
3. **Completeness** $\Rightarrow$ each basket lies on only one indifference curve.

Monotonicity



Cannot cross



Suppose that B preferred to A.
...but by definition of IC,
B indifferent to C
A indifferent to C
$\Rightarrow$ B indifferent to C by transitivity.
And thus a contradiction.

Indifference Curves - Example 1

Example

$$U = xy^2$$

Check that underlying preferences are complete, transitive, and monotonic.

Indifference Curves - Example 1

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$$MU_x = y^2;\ MU_y = 2xy;\ \text{for } U = 144$$

Indifference Curves - Example 1

Example

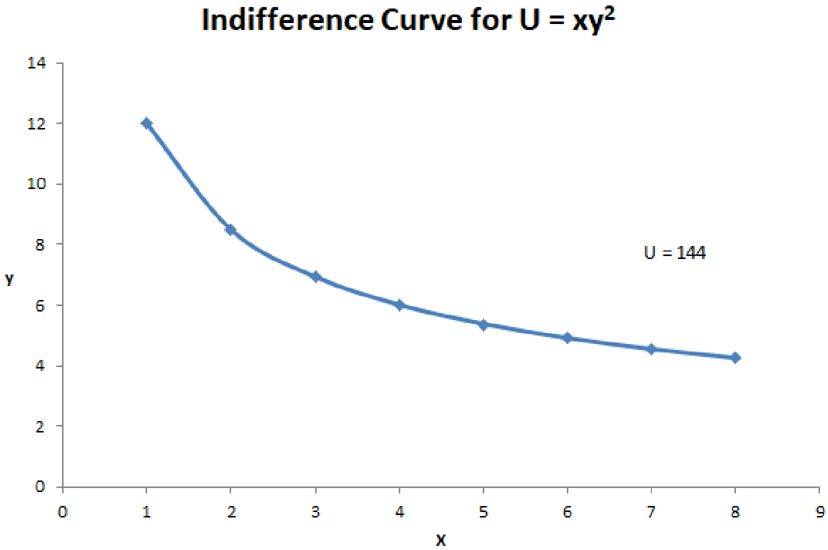
$$U = xy^2$$

Check that underlying preferences are complete, transitive, and monotonic.

$$MU_x = y^2;\ MU_y = 2xy;\ \text{for } U = 144$$

x	y	xy^2
8	4.24	143.8
4	6	144
3	6.93	144.07
1	12	144

Indifference Curves - Example 2



Marginal Rate of Substitution

Marginal Rate of Substitution

The **marginal rate of substitution**: is the maximum rate at which the consumer would be willing to substitute a little more of good x for a little less of good y;

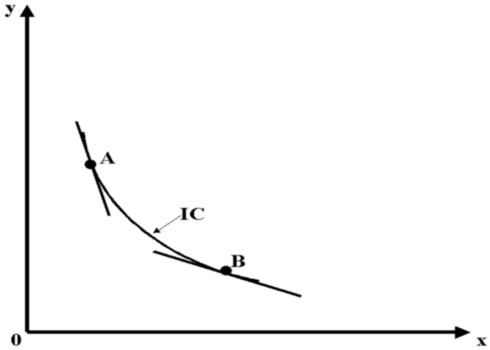
It is the increase in good x that the consumer would require in exchange for a small decrease in good y in order to leave the consumer just indifferent between consuming the old basket or the new basket;

It is the rate of exchange between goods x and y that does not affect the consumer's welfare;

It is the negative of the slope of the indifference curve:

$$MRS_{x,y} = -\frac{\Delta y}{\Delta x} \text{ for a constant level of preference}$$

The Diminishing Marginal Rate of Substitution



If the more of good x you have, the more you are willing to give up to get a little of good y or the indifference curves get flatter as we move out along the horizontal axis and steeper as we move up along the vertical axis.

Marginal Rate of Substitution

$$MU_x(\Delta x) + MU_y(\Delta y) = 0 \text{ ...along an IC...}$$

$$MU_x/MU_y = -\Delta y/\Delta x = MRS_{x,y}$$

Positive marginal utility implies the indifference curve has a negative slope (*implies monotonicity*).

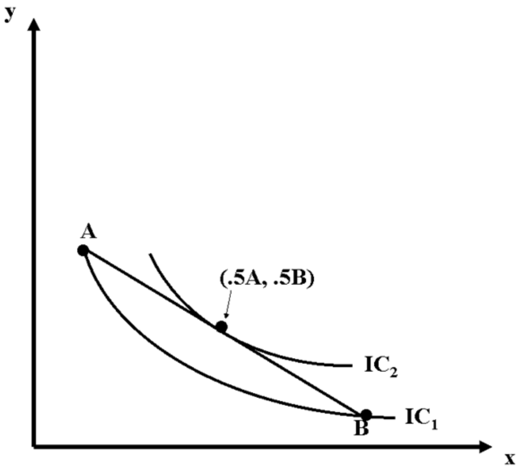
Diminishing marginal utility implies the indifference curves are convex to the origin (*implies averages preferred to extremes*).

Marginal Rate of Substitution

Implications of this substitution:

- Indifference curves are negatively-sloped, bowed out from the origin, preference direction is up and right.
- Indifference curves do not intersect the axes.

Key Property



Averages preferred to extremes ⇒ indifference curves are bowed toward the origin (convex to the origin).

Indifference Curves

Do the indifference curves intersect the axes?

A value of $x = 0$ or $y = 0$ is inconsistent with any positive level of utility.

Indifference Curves

Example

$$U = (xy)^{\frac{1}{2}};\ MU_x = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}} =;\ MU_y = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}$$

1. Is more better for both goods?

Indifference Curves

Example

$$U = (xy)^{\frac{1}{2}};\ MU_x = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}} =;\ MU_y = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}}$$

1. Is more better for both goods?
   Yes, since marginal utilities are positive for both.

Indifference Curves

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1. Is more better for both goods?
   Yes, since marginal utilities are positive for both.
2. Are the marginal utility for x and y diminishing?

Indifference Curves

Example

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   Yes. (For example, as x increases, for y constant, MU_x falls.)

Indifference Curves

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1. Is more better for both goods?
   Yes, since marginal utilities are positive for both.
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3. What is the marginal rate of substitution of x for y?

Indifference Curves

Example

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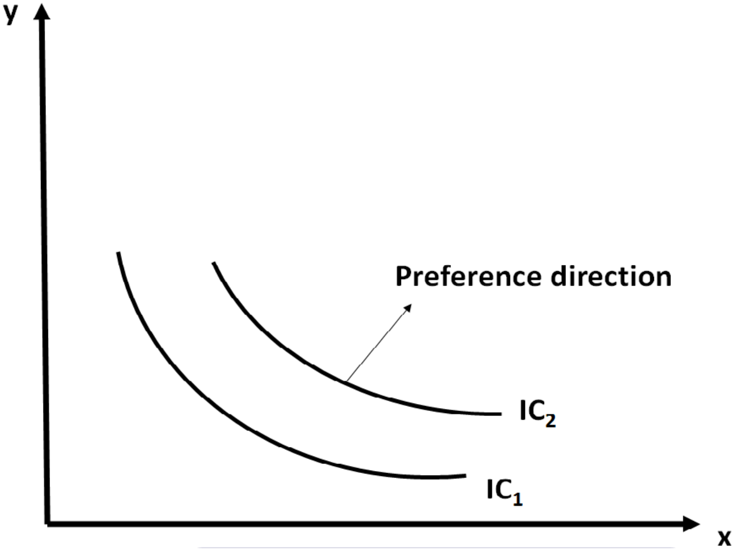
   Yes, since marginal utilities are positive for both.
2. Are the marginal utility for x and y diminishing?
   Yes. (For example, as x increases, for y constant, MU_x falls.)
3. What is the marginal rate of substitution of x for y?

$$MRS_{x,y} = MU_x/MU_y = y/x$$

Indifference Curves



Special Functional Forms

Cobb-Douglas

$$U(x, y) = Ax^{\alpha}y^{\beta}$$

where $\alpha + \beta = 1$; A, α, β are positive constants

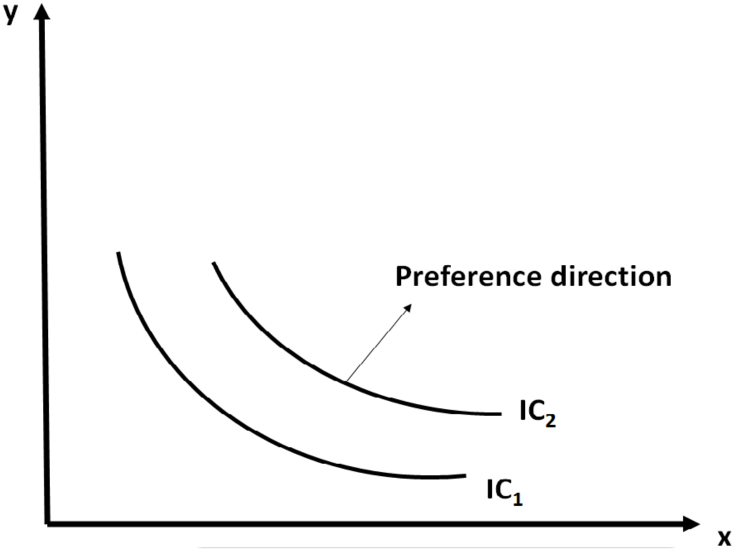
$$MU_x = \alpha A x^{\alpha-1} y^{\beta}$$

$$MU_y = \beta A x^{\alpha} y^{\beta-1}$$

$$MRS_{x,y} = \frac{\alpha y}{\beta x}$$

Standard case

Special Functional Forms



Special Functional Forms

Perfect Substitutes

$$U = Ax + By$$

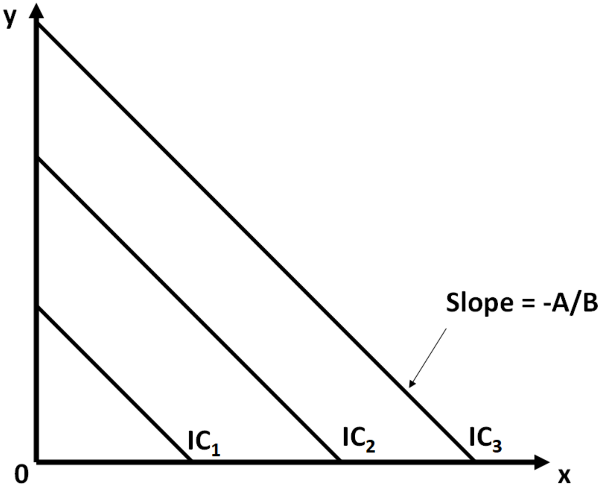
Where A and B are positive constants

$$MU_x = A$$

$$MU_y = B$$

$MRS_{x,y} = \dfrac{A}{B}$ so that 1 unit of x is equal to $\dfrac{B}{A}$ units of y *everywhere* (constant MRS).

Special Functional Forms



Special Functional Forms

Perfect Complements

$$U = A\min(x, y)$$

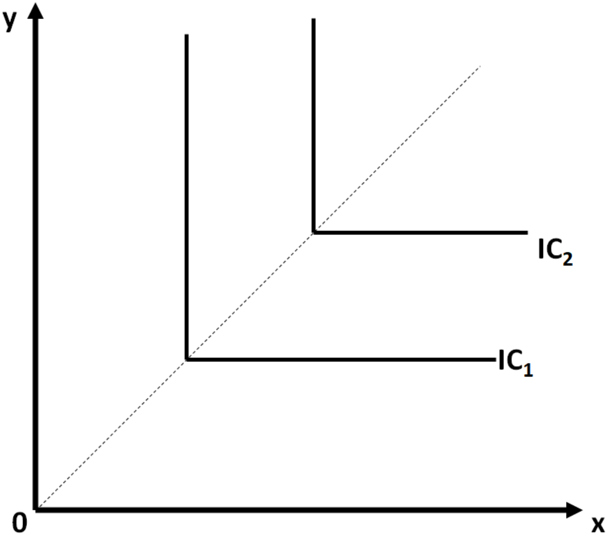
Where A is a positive constant

$$MU_x = 0 \text{ or } A$$

$$MU_y = 0 \text{ or } A$$

$$MRS_{x,y} \text{ is 0 or infinite or undefined (corner).}$$

Special Functional Forms



Special Functional Forms

Quasi-linear preferences

$$U = v(x) + Ay$$

Where A is a positive constant

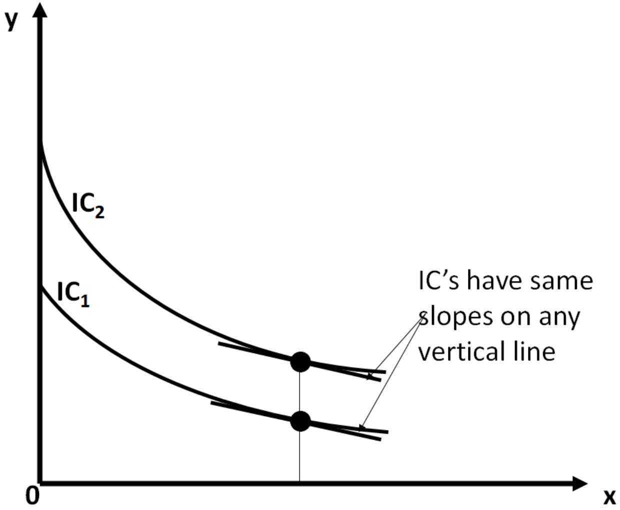
$$MU_x = v'(x) = \frac{\Delta V(x)}{\Delta x}$$

$$MU_y = A$$

"The only thing that determines your personal trade-off between x and y is how much x you already have."

It can be used to "add up" utilities across individuals.

Special Functional Forms



Consumer Choice

Adapted from Chapter 4 of Besanko's Microeconomics

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April 4, 2025

Overview

1. The Budget Constraint

2. Consumer Choice

Key Definitions

Budget Set

The set of baskets that are affordable.

Budget Constraint

The set of baskets that the consumer may purchase given the limits of the available income.

Budget Line

The set of baskets that one can purchase when spending all available income.

$$p_x \cdot x + p_y \cdot y = I$$

$$y = \frac{I}{p_y} + \frac{p_x}{p_y}$$

The Budget Constrain

Assume only two goods available: x and y

Price of x: p_x

Price of y: p_y

Income: I

Total expenditure on basket $(x, y) : p_x \cdot x + p_y \cdot y$

The Basket is Affordable if total expenditure does not exceed total Income:

$$p_x \cdot x + p_y \cdot y \leq I$$

A Budget Constraint Example

Example

Two goods available: x and y

$$I = \$10$$
$$p_x = \$1$$
$$p_y = \$2$$

A Budget Constraint Example

Example

Two goods available: x and y

$$I = \$10$$

$$p_x = \$1$$

$$p_y = \$2$$

All income spent on x

A Budget Constraint Example

Example

Two goods available: x and y

$I = \$10$

$p_x = \$1$

$p_y = \$2$

All income spent on $x \rightarrow \frac{I}{p_x}$ units of x bought

A Budget Constraint Example

Example

Two goods available: x and y

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All income spent on $x \rightarrow \frac{I}{p_x}$ units of x bought

All income spent on y

A Budget Constraint Example

Example

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A Budget Constraint Example

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Budget Line:

A Budget Constraint Example

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All income spent on $y \rightarrow \frac{I}{p_y}$ units of y bought

Budget Line: $1x + 2y = 10$ or $y = 5 - \frac{1}{2}x$

A Budget Constraint Example

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Slope of Budget Line:

A Budget Constraint Example

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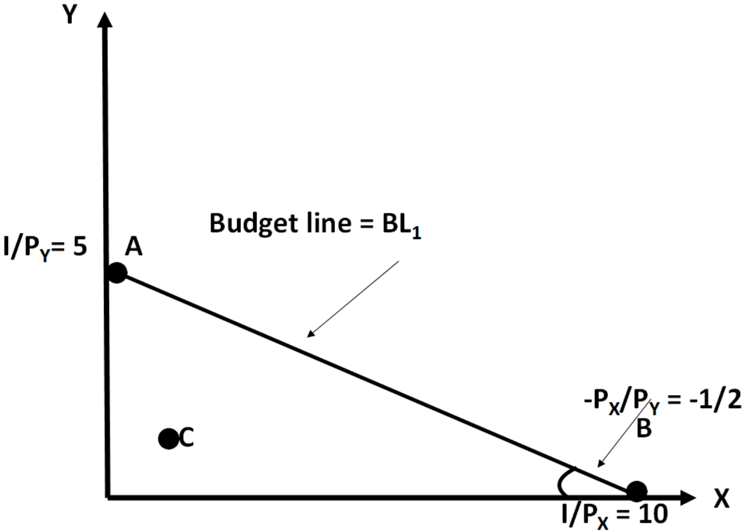
Slope of Budget Line: $= -\frac{p_x}{p_y} = -\frac{1}{2}$

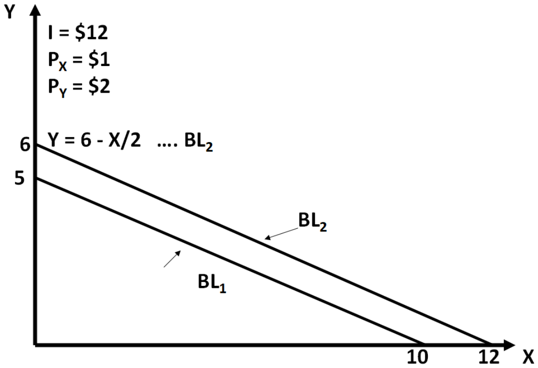
A Budget Constraint Example



The Budget Constraint

- Location of budget line shows what the income level is.
- Increase in Income will shift the budget line to the right.
  $\rightarrow$ More of each product becomes affordable.
- Decrease in Income will shift the budget line to the left.
  $\rightarrow$ Less of each product becomes affordable.

A Budget Constraint Example



Shift of a budget line

- If income rises, the budget line shifts parallel to the right (shifts out)
- If income falls, the budget line shifts parallel to the left (shifts in)

The Budget Constraint

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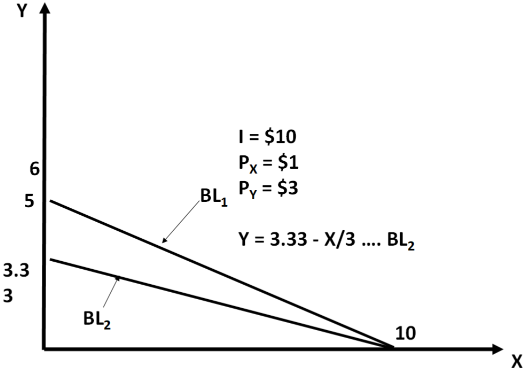

Rotation of a budget line

- If the price of x rises, the budget line gets steeper and the horizontal intercept shifts in.
- If the price of x falls, the budget line gets flatter and the horizontal intercept shifts out.

Another Budget Constraint Example

Example

Two goods available: x and y

$$I = \$800$$
$$p_x = \$20$$
$$p_y = \$40$$

Another Budget Constraint Example

Example

Two goods available: x and y

$$I = \$800$$

$$p_x = \$20$$

$$p_y = \$40$$

All income spent on x

Another Budget Constraint Example

Example

Two goods available: x and y

$I = \$800$

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All income spent on $x \rightarrow \frac{I}{p_x}$ units of x bought

Another Budget Constraint Example

Example

Two goods available: x and y

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All income spent on y

Another Budget Constraint Example

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Two goods available: x and y

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Another Budget Constraint Example

Example

Two goods available: x and y

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Budget Line:

Another Budget Constraint Example

Example

Two goods available: x and y

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All income spent on $y \rightarrow \frac{I}{p_y}$ units of y bought

Budget Line: $20x + 40y = 800$ or $y = 20 - \frac{1}{2}x$

Another Budget Constraint Example

Example

Two goods available: x and y

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Budget Line: $20x + 40y = 800$ or $y = 20 - \frac{1}{2}x$

Slope of Budget Line:

Another Budget Constraint Example

Example

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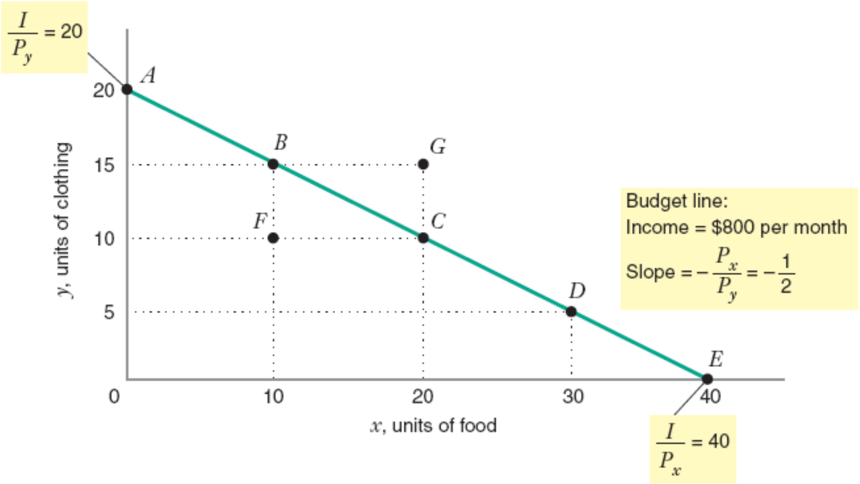
Slope of Budget Line: $= -\frac{p_x}{p_y} = -\frac{1}{2}$

Another Budget Constraint Example



Consumer Choice

Assume:

- Only non-negative quantities.
- *Rational* choice: The consumer chooses the basket that maximizes his satisfaction given the constraint that his budget imposes.

Consumer's Problem

$$\max u(x, y)$$

$$s.t.\ p_x x + p_y y \leq I$$

Interior Optimum

Interior Optimum

Interior Optimum is the optimal consumption basket is at a point where the indifference curve is just *tangent* to the budget line.

Tangent

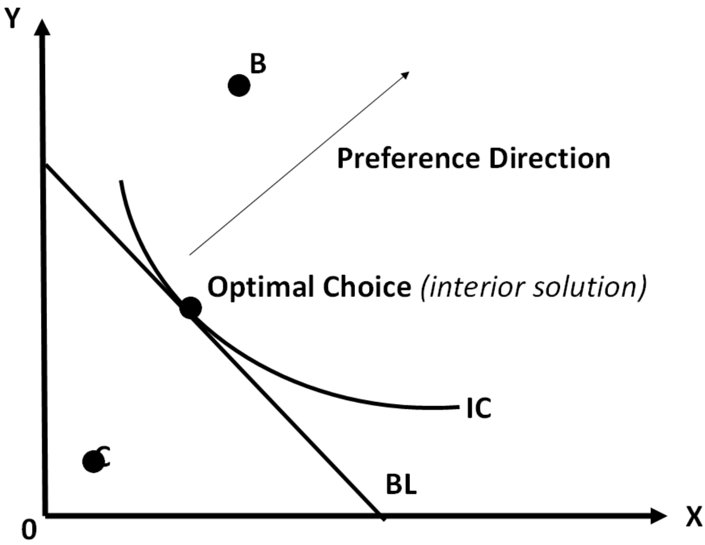
A **tangent** to a function is a straight line that has the same slope as the function.

Therefore:

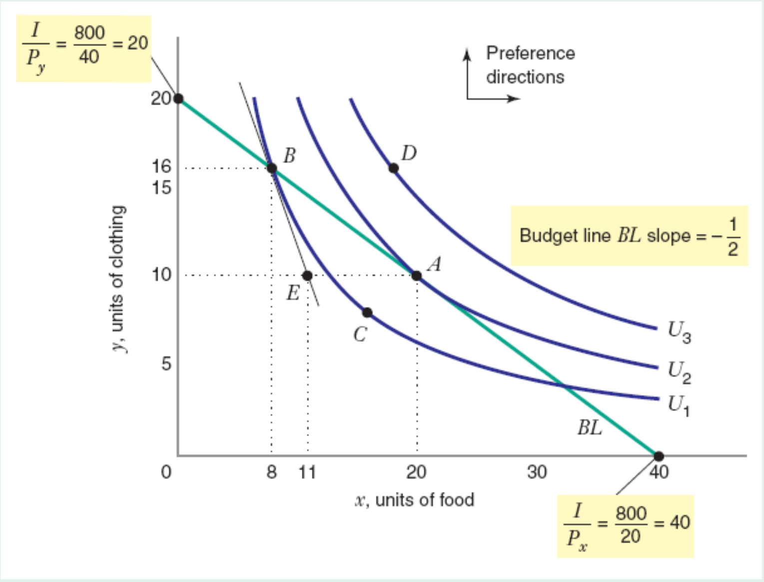
$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

The rate at which the consumer would be willing to exchange x for y is the same as the rate at which they are exchanged in the *marketplace*.

Interior Consumer Optimum



Interior Consumer Optimum



Interior Consumer Optimum

Example

Assumptions:

$$U(x, y) = xy$$

$$MU_x = y \text{ and } MU_y = x$$

$$I = \$1000$$

$$p_x = 50 \text{ and } p_y = 200$$

Basket A contains: $(x = 4; y = 4)$

Basket B contains: $(x = 10; y = 2.5)$

Interior Consumer Optimum

Example

Assumptions:

$$U(x, y) = xy$$

$$MU_x = y \text{ and } MU_y = x$$

$$I = \$1000$$

$$p_x = 50 \text{ and } p_y = 200$$

Basket A contains: $(x = 4; y = 4)$

Basket B contains: $(x = 10; y = 2.5)$

Is either basket the optimal choice for the consumer?

Interior Consumer Optimum

Example

$$\text{Basket } A\text{: } MRS_{x,y} = MU_x / MU_y = \frac{y}{x} = \frac{4}{4} = 1$$

Interior Consumer Optimum

Example

$$\text{Basket } A\text{: } MRS_{x,y} = MU_x / MU_y = \frac{y}{x} = \frac{4}{4} = 1$$

$$\text{Slope of budget line: } -\frac{p_x}{p_y} = -\frac{1}{4}$$

Interior Consumer Optimum

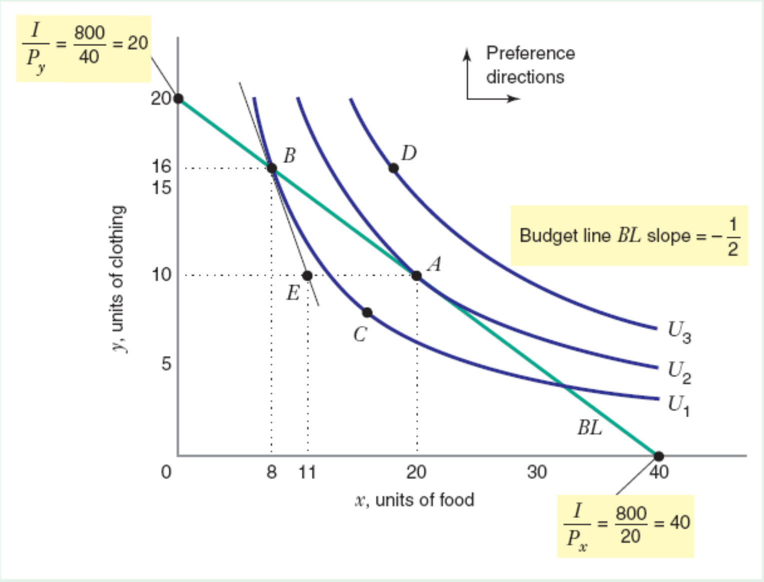
Example

$$\text{Basket } A\text{: } MRS_{x,y} = MU_x / MU_y = \frac{y}{x} = \frac{4}{4} = 1$$

$$\text{Slope of budget line: } -\frac{p_x}{p_y} = -\frac{1}{4}$$

$$\text{Basket } B\text{: } MRS_{x,y} = MU_x / MU_y = \frac{y}{x} = \frac{1}{4} = 4$$

Interior Consumer Optimum



Equal Slope Condition

Bang for the buck

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

At the optimal basket, each good gives equal bang for the buck

Now, we have two equations to solve for two unknowns (quantities of x and y in the optimal basket):

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

$$p_x x + p_y y = I$$

Constrained Optimization

Exercise 1

$$U(F, C) = FC$$

$$p_F = \$1$$

$$p_C = \$2$$

$$I = \$12$$

Solve for optimal choice of food and clothing

Some concepts

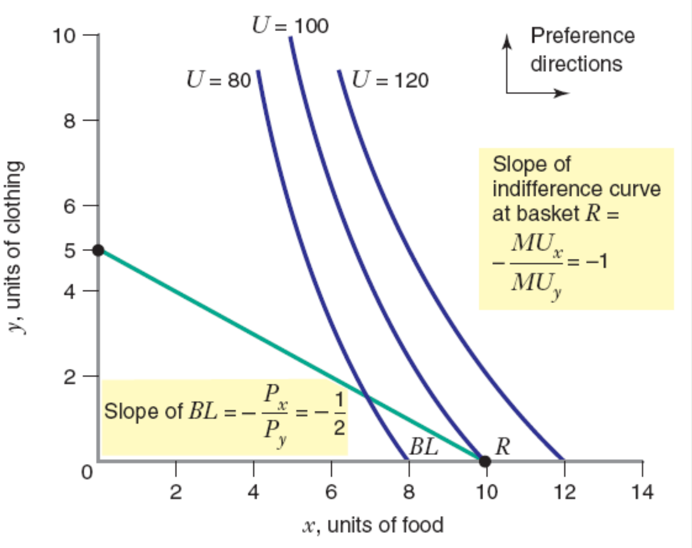
Corner Points

One good is not being consumed at all – Optimal basket lies on the axis.

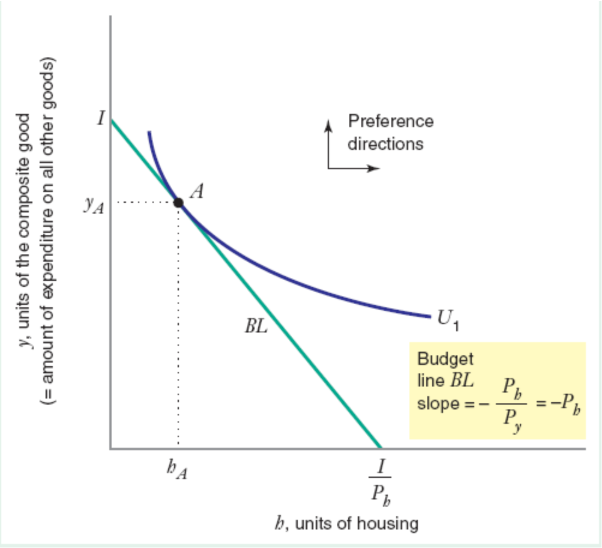
Composite Goods

A good that represents the collective expenditure on every other good except the commodity being considered.

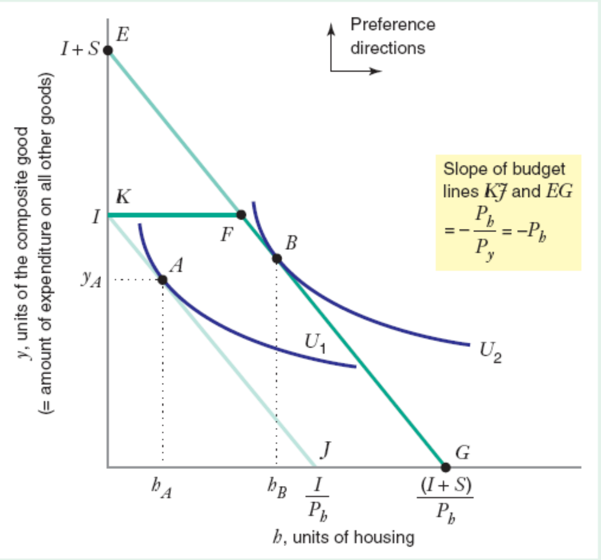
Corner Solutions



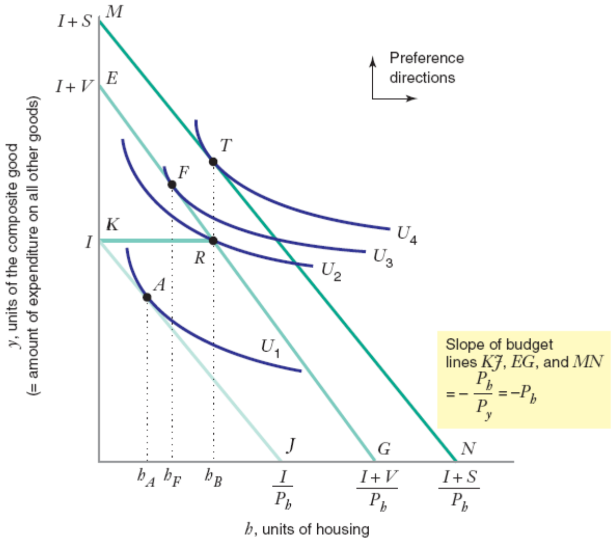
Composite Goods



Composite Goods



Composite Goods



The Theory Of Demand

Adapted from Chapter 5 of Besanko's Microeconomics

Luisa Lorè
Department of Economics
University of Innsbruck

April 9, 2025
April 11, 2025

Overview

1. Individual Demand Curves

2. Income and Substitution Effects & the Slope of Demand

3. Constructing Market Demand

The aim of this chapter is to study the **Effects of a Change in Price**

- Optimal Choice
- Demand Curve

Individual Demand Curves

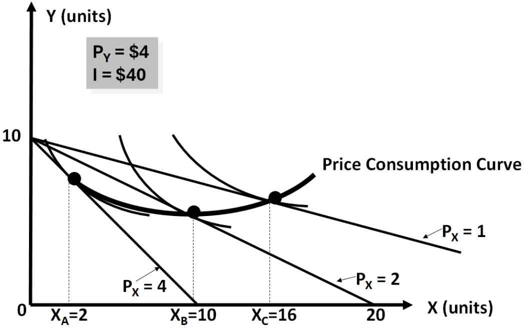
- In Chapter 4, consumer's optimal basket was determined.
- Thus, we can tell – for a given income and prices of other goods – how much a consumer will demand of x for a given price of x.
- This is a point on the consumer's demand curve.
- We can find more points on the demand curve for x by changing the price of x and determining how much of x the consumer will demand – prices of other goods and income are held constant.

Individual Demand Curves

Price Consumption Curve of Good x

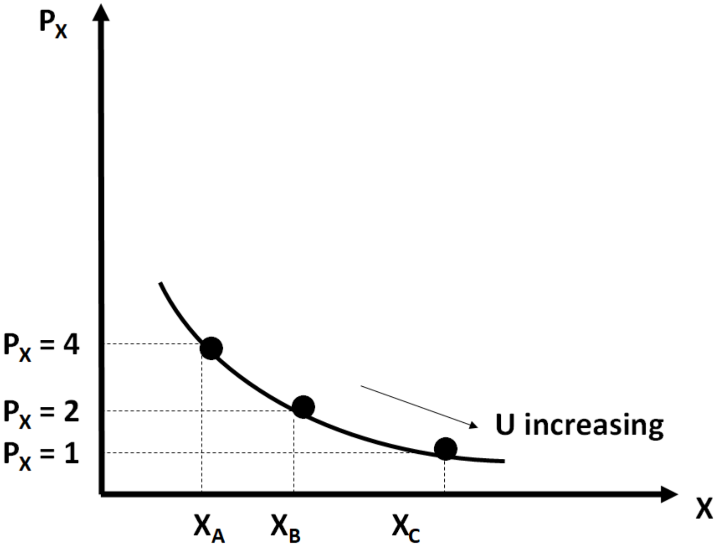
The Price Consumption Curve of Good x is the set of optimal baskets for every possible price of good x, holding all other prices and income constant.

Price Consumption Curve



The price consumption curve for good x can be written as the quantity consumed of good x for any price of x. This is the individual's demand curve for good x.

Individual Demand Curve for x



Key Points

- The consumer is maximizing utility at every point along the demand curve.
- The marginal rate of substitution falls along the demand curve as the price of x falls (if there was an interior solution).
- As the price of x falls, it causes the consumer to move down and to the right along the demand curve as utility increases in that direction.
- The demand curve is also the "willingness to pay" curve – and willingness to pay for an additional unit of x falls as more x is consumed.

Demand Curve for the good x

Algebraically, we can solve for the individual's demand using the following equations:

1. $p_x x + p_y y = I$
2. $\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$

If this never holds, a corner point may be substituted where $x = 0$ or $y = 0$.

Demand Curve with an Interior Solution

Suppose that $U(x,y) = xy$. $MU_x = y$ and $MU_y = x$. The prices of x and y are p_x and p_y, respectively and income $= I$.
We have:

1. $p_x x + p_y y = I$
2. $\frac{x}{p_x} = \frac{y}{p_y}$

Substituting the second condition into the budget constraint, we then have:

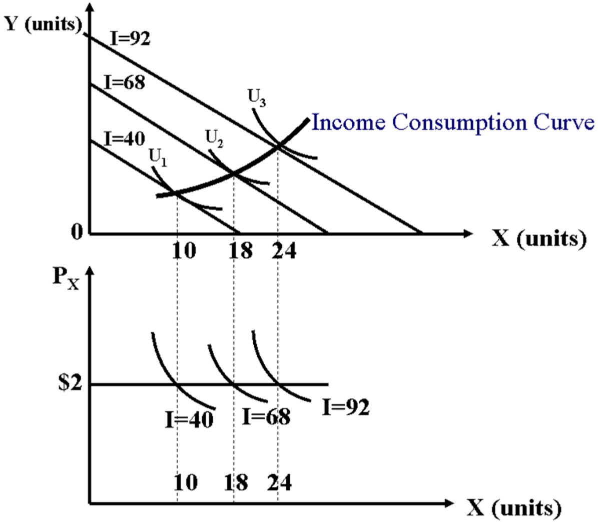
$$p_x x + p_y \frac{p_x}{p_y} x = I \text{ or } x = \frac{1}{2}\frac{I}{p_x}$$

Change in Income & Demand

Income Consumption Curve

The **income consumption curve of good x** is the set of optimal baskets for every possible level of income.
We can graph the points on the income consumption curve as points on a shifting demand curve.

Income Consumption Curve

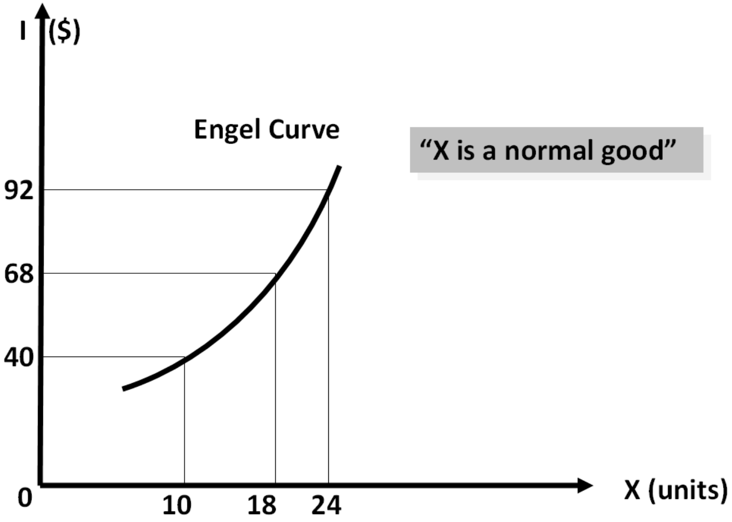


Engel Curves

Engel Curve

The income consumption curve for good x also can be written as the quantity consumed of good x for any income level. This is the individual's **Engel Curve** for good x. When the income consumption curve is positively sloped, the slope of the Engel Curve is positive.

Engel Curves



Definitions of Goods

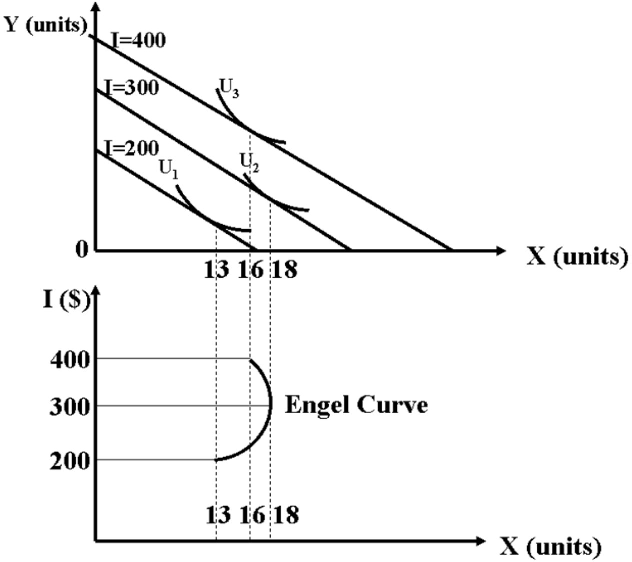
Normal Good

If the income consumption curve shows that the consumer purchases more of good x as her income rises, good x is a **normal good**.
Equivalently, if the slope of the Engel curve is positive, the good is a normal good.

Inferior Good

If the income consumption curve shows that the consumer purchases less of good x as her income rises, good x is an **inferior good**.
Equivalently, if the slope of the Engel curve is negative, the good is an inferior good.

Definitions of Goods



Backward Bending Engel Curve
– a good can be normal over some ranges and inferior over others

Impact of Change in the Price of a Good

Substitution Effect

Relative change in price affects the amount of good that is bought as consumer tries to achieve the same level of utility.

Income Effect

Consumer's purchasing power changes and affects the consumer in a way similar to effect of a change in income.

The Substitution Effect

- As the price of x falls, all else constant, good x becomes cheaper relative to good y.
- This change in relative prices alone causes the consumer to adjust his/ her consumption basket.
- This effect is called the substitution effect.
- The substitution effect always is negative.
- Usually, a move along a demand curve will be composed of both effects.

Impact of Change in the Price of a Good

Income Effect

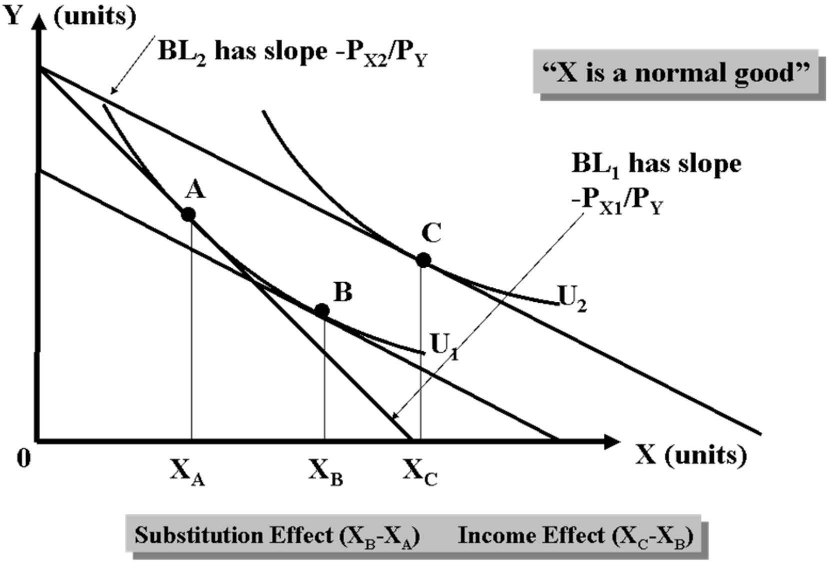
As the price of x falls, all else constant, purchasing power rises. As the price of x rises, all else constant, purchasing power falls.

The income effect may be positive (normal good) or negative (inferior good).

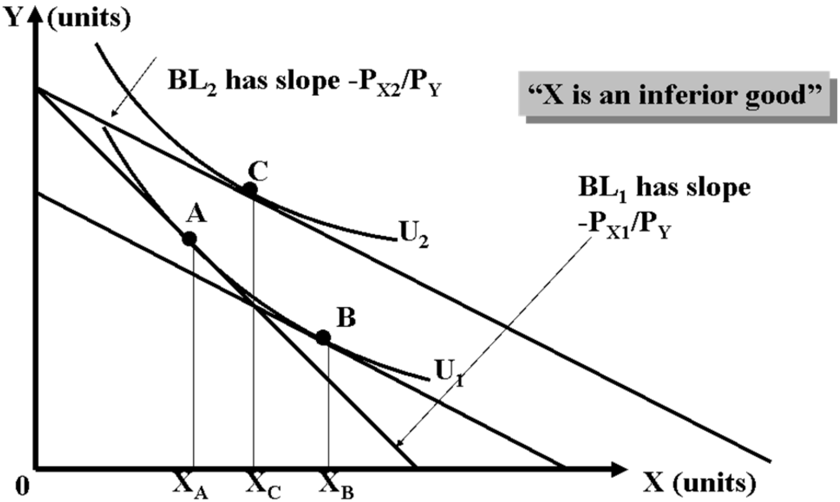
Impact of Change in the Price of a Good

- If price of a good falls – consumer substitutes into the good to achieve the same level of utility.
- When price falls – purchasing power increases the consumer can buy the same amount and still have money left.

The Substitution and Income Effects



The Substitution and Income Effects



Substitution Effect (X_B-X_A)   Income Effect (X_C-X_B)

Giffen Goods

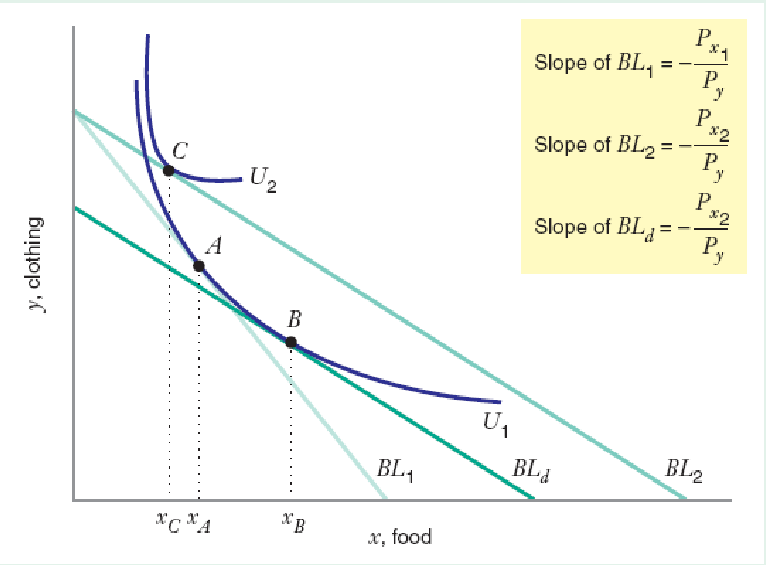
Giffen Good

If a good is so inferior that the net effect of a price decrease of good x, all else constant, is a decrease in consumption of good x, good x is a **Giffen good**.

For Giffen goods, demand does not slope down.

When might an income effect be large enough to offset the substitution effect? The good would have to represent a very large proportion of the budget.

Giffen Goods - Income and Substitution Effects



Income and Substitution Effects - Example 1

Example

Suppose $U(x, y) = xy \to MU_x = y,\ MU_y = x$

$p_y = \$1,\ I = \$72$

Suppose that $p_{x1} = \$9$.

What is the (initial) optimal consumption basket?

Income and Substitution Effects - Example 1

Example

Suppose $U(x,y) = xy \rightarrow MU_x = y, \; MU_y = x$

$$p_y = \$1, \; I = \$72$$

Suppose that $p_{x1} = \$9$.

What is the (initial) optimal consumption basket?

Tangency Condition: $\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \rightarrow y = 9x$

Constraint: $p_x x + p_y y = I \rightarrow 9x + y = 72$

Income and Substitution Effects - Example 1

Example

Suppose $U(x,y) = xy \rightarrow MU_x = y,\ MU_y = x$

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Constraint: $p_x x + p_y y = I \rightarrow 9x + y = 72$

Solving: $x_1 = 4$ and $y_1 = 36$

Income and Substitution Effects - Example 2

Example

$$\text{Suppose } U(x, y) = xy \rightarrow MU_x = y,\ MU_y = x$$

$$p_y = \$1,\ I = \$72$$

Suppose that price of x falls and $p_{x2} = \$4$.

What is the (final) optimal consumption basket?

Income and Substitution Effects - Example 2

Example

Suppose $U(x, y) = xy \rightarrow MU_x = y,\ MU_y = x$

$p_y = \$1,\ I = \$72$

Suppose that price of x falls and $p_{x2} = \$4$.

What is the (final) optimal consumption basket?

Tangency Condition: $\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \rightarrow y = 4x$

Constraint: $p_x x + p_y y = I \rightarrow 4x + y = 72$

Income and Substitution Effects - Example 2

Example

Suppose $U(x, y) = xy \rightarrow MU_x = y,\ MU_y = x$

$p_y = \$1,\ I = \$72$

Suppose that price of x falls and $p_{x2} = \$4$.

What is the (final) optimal consumption basket?

Tangency Condition: $\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \rightarrow y = 4x$

Constraint: $p_x x + p_y y = I \rightarrow 4x + y = 72$

Solving: $x_2 = 9$ and $y_2 = 36$

Income and Substitution Effects - Example 3

Example

Find the decomposition basket B.

1. It must lie on the *original* indifference curve U_1 along with basket $A \rightarrow U_1 = xy = 4 \cdot 36 = 144$.
2. It must lie at the point where the decomposition budget line is tangent to the indifference curve.
3. Price of x (p_x) on the decomposition budget line is final price of \$4.

Example

Find the decomposition basket B.

1. It must lie on the *original* indifference curve U_1 along with basket $A \rightarrow U_1 = xy = 4 \cdot 36 = 144$.
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Tangency Condition: $\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \rightarrow y = 4x$

Combined with $xy = 144 \rightarrow x_B = 6, y_B = 24$

Example

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Tangency Condition: $\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \rightarrow y = 4x$

Combined with $xy = 144 \rightarrow x_B = 6, y_B = 24$

Substitution Effect: $x_B - x_1 = 6 - 4 = 2$ units of x

Income Effect: $x_2 - x_B = 9 - 6 = 3$ units of x

Consumer Surplus

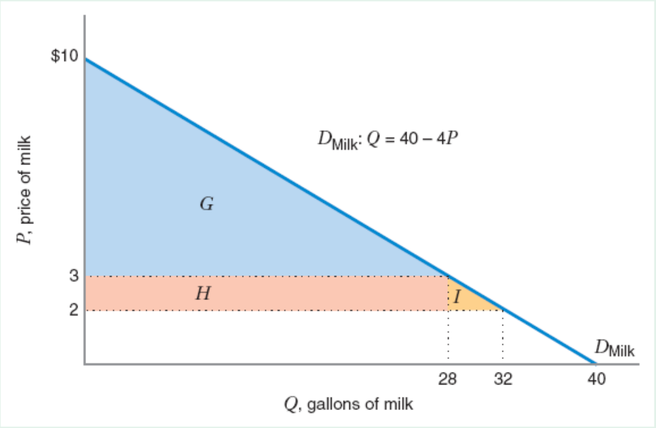
- The individual's demand curve can be seen as the individual's **willingness to pay** curve.
- On the other hand, the individual must only **actually** pay the market price for (all) the units consumed.
- Consumer Surplus is the difference between what the consumer is willing to pay and what the consumer actually pays.

Consumer Surplus

Consumer Surplus

The net economic benefit to the consumer due to a purchase (i.e. the willingness to pay of the consumer net of the actual expenditure on the good) is called **consumer surplus**. The area **under** an ordinary demand curve and **above** the market price provides a measure of consumer surplus

Consumer Surplus



$$G = \frac{1}{2}(10 - 3)(28) = 98$$

$$H + I = 28 + 2$$

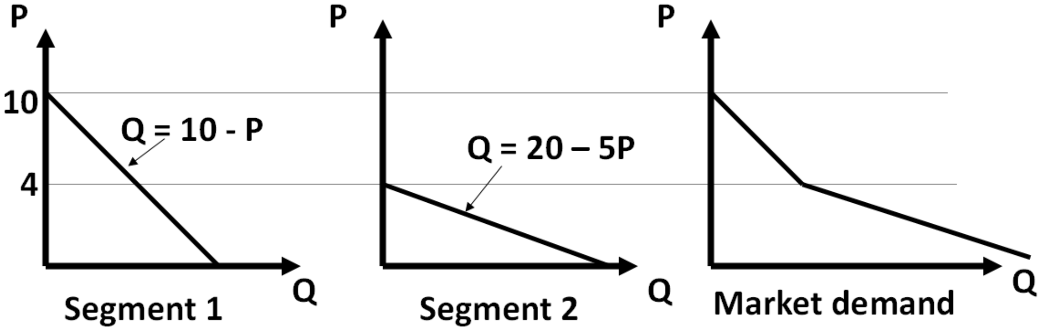
$$CS_2 = \frac{1}{2}(10 - 2)(32) = 128$$

$$CS_P = (10 - P)(40 - 4P)$$

Market Demand

The market demand function is the horizontal sum of the individual (or segment) demands.
In other words, market demand is obtained by adding the quantities demanded by the individuals (or segments) at each price and plotting this total quantity for all possible prices.

Market Demand



Network Externalities

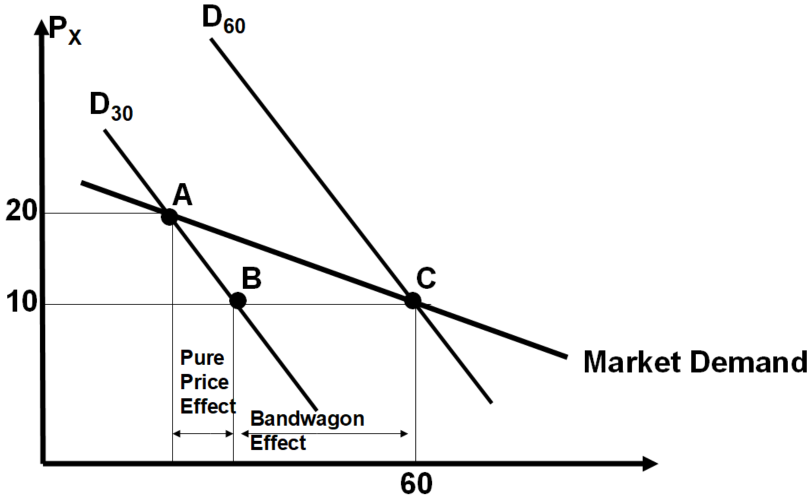
If one consumer's demand for a good changes with the number of other consumers who buy the good, there are **network externalities.**

Network Externalities

Bandwagon effect

The **Bandwagon effect** is a positive network externality that refers to the increase in each consumer's demand for a good as more consumers buy the good.
(Increased quantity demanded when more consumers purchase)

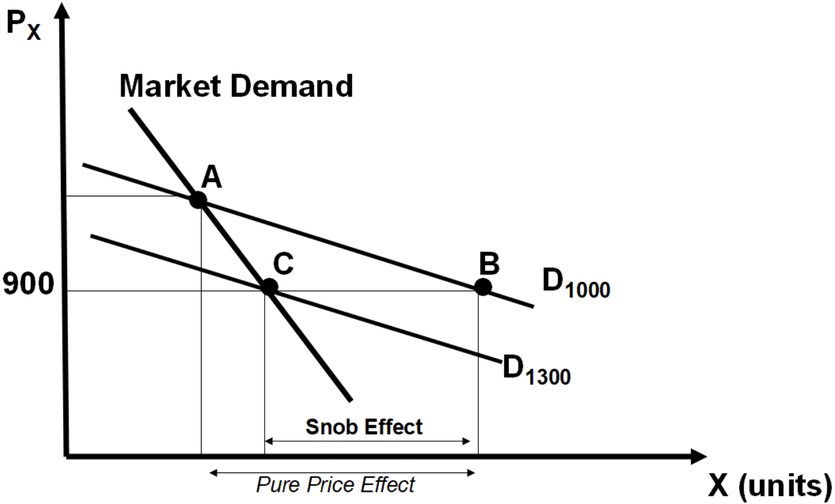
Network Externalities



Snob effect

The **Snob effect** is a negative network externality that refers to the decrease in each consumer's demand as more consumers buy the good.
(Decreased quantity demanded when more consumers purchase)

Network Externalities



Inputs and Production Functions

Adapted from Chapter 6 of Besanko's Microeconomics

Luisa Lorè
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University of Innsbruck

May 2, 2025

Overview

Key Concepts

Productive resources, such as labor and capital equipment, that firms use to manufacture goods and services are called **inputs or factors of production**.

The amount of goods and services produces by the firm is the firm's **output**.

Production transforms a set of inputs into a set of outputs.

Technology determines the quantity of output that is feasible to attain for a given set of inputs.

Key Concepts

Production function

The production function tells us the maximum possible output that can be attained by the firm for any given quantity of inputs.

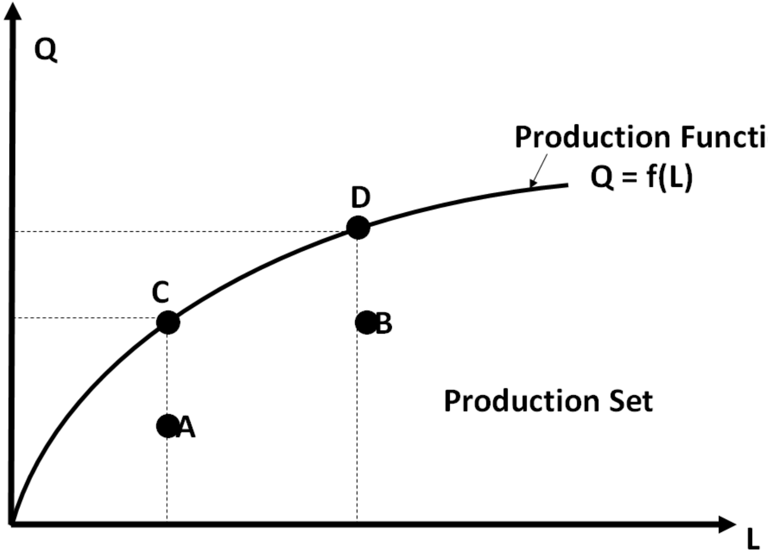
$$Q = f(K, L)$$

$$\begin{aligned} Q &= \text{Output} \\ K &= \text{Capital} \\ L &= \text{Labor} \end{aligned}$$

Production set

The **production set** is a set of technically feasible combinations of inputs and outputs.

The Production Function & Technical Efficiency



The Production Function & Technical Efficiency

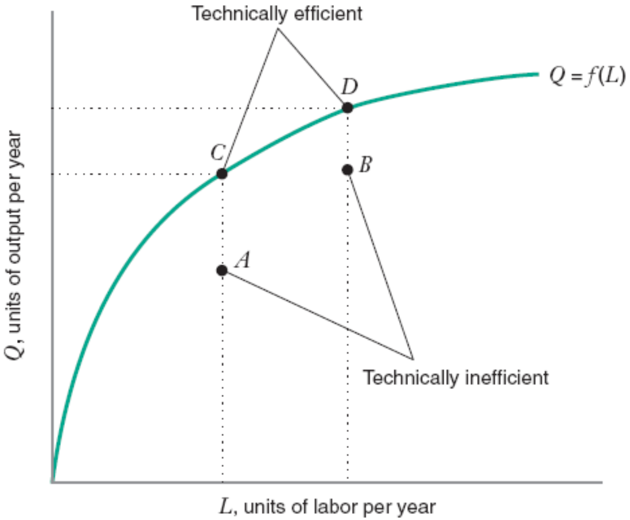
Technically efficient: Sets of points in the production function that maximizes output given input (labor).

$$Q = f(K, L)$$

Technically inefficient: Sets of points that produces less output than possible for a given set of input (labor).

$$Q < f(K, L)$$

The Production Function & Technical Efficiency



Labor Requirements Function

Labor requirements function

$$L = g(Q)$$

Labor Requirements Function

Labor requirements function

$$L = g(Q)$$

Example

From the Labor requirements function

$$L = Q^2$$

Labor Requirements Function

Labor requirements function

$$L = g(Q)$$

Example

From the Labor requirements function

$$L = Q^2$$

To the Production function

$$Q = \sqrt{L}$$

The Production & the Utility Functions

Production Function	Utility Function
Output from inputs	Preference level from purchases
Derived from technologies	Derived from preferences
Cardinal[1]	Ordinal
Marginal Product	Marginal Utility
Isoquant[2]	Indifference Curve
Marginal Rate of Technical Substitution	Marginal Rate of Substitution

[1]given amount of inputs yields a unique and specific amount of output

[2]all possible combinations of inputs that just suffice to produce a given amount of output

The Production Function & Technical Efficiency

Production Function $Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$ in Table Form

L \ K	0	10	20	30	40	50
0	0	0	0	0	0	0
10	0	10	14	17	20	22
20	0	14	20	24	28	32
30	0	17	24	30	35	39
40	0	20	28	35	40	45
50	0	22	32	39	45	50

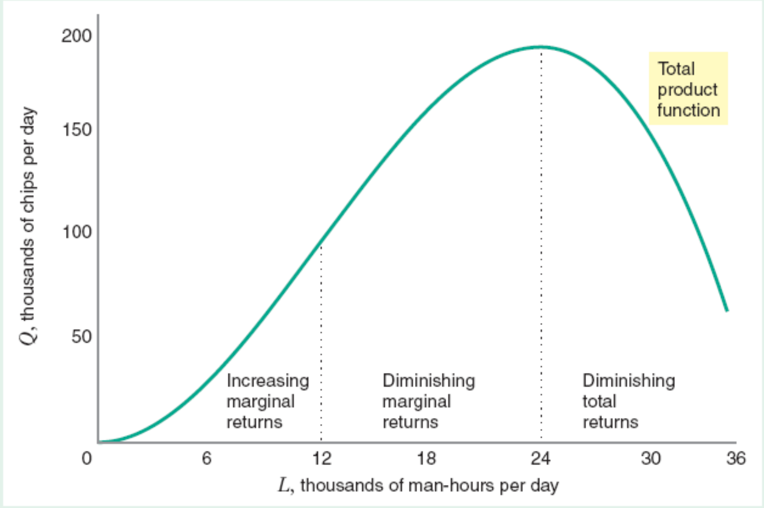
Total Product

Total Product Function: A single-input production function. It shows how total output depends on the level of the input

Marginal Return to Labor

- **Increasing Marginal Returns to Labor**: An increase in the quantity of labor increases total output at an increasing rate.
- **Diminishing Marginal Returns to Labor**: An increase in the quantity of labor increases total output but at a decreasing rate.
- **Diminishing Total Returns to Labor**: An increase in the quantity of labor decreases total output.

Total Product



The Marginal Product

Marginal Product

The **marginal product** of an input is the change in output that results from a small change in an input *holding the levels of all other inputs constant*.

$$MP_L = \frac{\Delta Q}{\Delta L}$$

$$MP_K = \frac{\Delta Q}{\Delta K}$$

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Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

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$$MP_L = \frac{\partial Q}{\partial L} =$$

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$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$
$$MP_L = \frac{\partial Q}{\partial L} = \frac{1}{2} K^{\frac{1}{2}} L^{-\frac{1}{2}}$$

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The Average Product

Average Product

The **average product** of an input is equal to the total output that is to be produced divided by the quantity of the input that is used in its production:

$$AP_L = \frac{Q}{L}$$

$$AP_K = \frac{Q}{K}$$

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Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

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Example

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Example

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$$AP_L = \frac{Q}{L} = K^{\frac{1}{2}} L^{-\frac{1}{2}}$$

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Average Product

The **average product** of an input is equal to the total output that is to be produced divided by the quantity of the input that is used in its production:

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Example

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$$AP_L = \frac{Q}{L} = K^{\frac{1}{2}} L^{-\frac{1}{2}}$$
$$AP_K = \frac{Q}{K} =$$

The Average Product

Average Product

The **average product** of an input is equal to the total output that is to be produced divided by the quantity of the input that is used in its production:

$$AP_L = \frac{Q}{L}$$

$$AP_K = \frac{Q}{K}$$

Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$
$$AP_L = \frac{Q}{L} = K^{\frac{1}{2}} L^{-\frac{1}{2}}$$
$$AP_K = \frac{Q}{K} = K^{-\frac{1}{2}} L^{\frac{1}{2}}$$

Diminishing Returns

Law of diminishing marginal returns

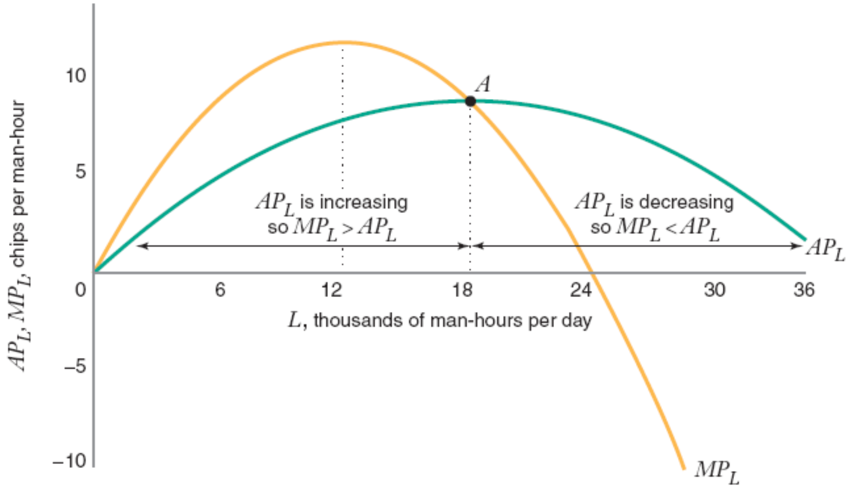
The **law of diminishing marginal** returns states that marginal products (*eventually*) decline as the quantity used of a single input increases.

Total, Average, and Marginal Products

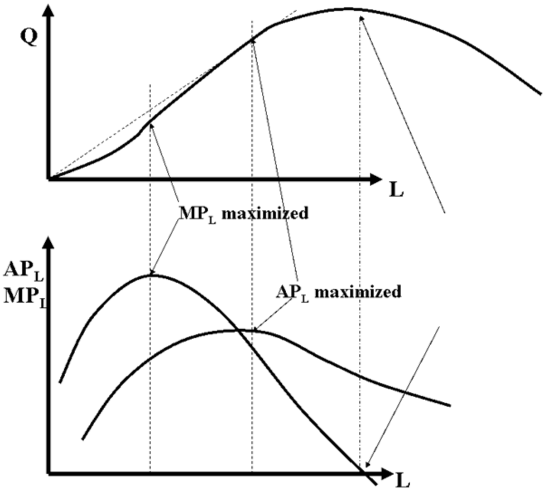
$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

L	Q	APL	MPL
6	30	5	-
12	96	8	11
18	162	9	11
24	162	9	11
30	150	5	-7

Total, Average, and Marginal Products



Total, Average, and Marginal Magnitudes



TP_L maximized where MP_L is zero.
TP_L falls where MP_L is negative;
TP_L rises where MP_L is positive.

Production functions with 2 inputs

Marginal product: Change in total product holding other inputs fixed.

$$MP_{I_1} = \frac{\text{Change in quantity of output, } Q}{\text{Change in quantity of one input, } I_1}\Big|_{I_2 \text{ is held constant}}$$

$$MP_L = \frac{\Delta Q}{\Delta L}\Big|_{K \text{ is held constant}}$$

$$MP_K = \frac{\Delta Q}{\Delta K}\Big|_{L \text{ is held constant}}$$

Isoquants

Isoquant

An **isoquant** traces out all the combinations of inputs (labor and capital) that allow that firm to produce the same quantity of output.

Isoquants

Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

What is the equation of the isoquant for $Q = 20$?

Isoquants

Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

What is the equation of the isoquant for $Q = 20$?

$$20 = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

$$400 = KL$$

$$K = \frac{400}{L}$$

Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

What is the equation of the isoquant for $Q = 20$?

$$20 = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

$$400 = KL$$

$$K = \frac{400}{L}$$

...and the isoquant for $Q = Q^*$?

Isoquants

Example

$$Q = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

What is the equation of the isoquant for $Q = 20$?

$$20 = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

$$400 = KL$$

$$K = \frac{400}{L}$$

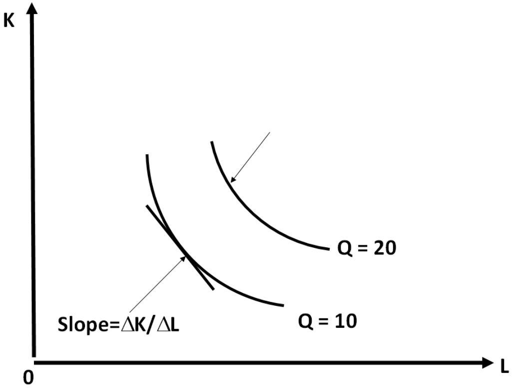
... and the isoquant for $Q = Q^*$?

$$Q^* = K^{\frac{1}{2}} L^{\frac{1}{2}}$$

$$Q^{*2} = KL$$

$$K = \frac{Q^{*2}}{L}$$

Isoquants



All combinations of (L, K) along the isoquant produce 20 units of output.

Marginal Rate of Technical Substitution

Marginal Rate of Technical Substitution

The **marginal rate of technical substitution** measures the amount of an input, L, the firm would require in exchange for using a little less of another input, K, in order to just be able to produce the same output as before.

$$MRTS_{L,K} = -\frac{\Delta K}{\Delta L}$$

Marginal products and the MRTS are related:

$$MP_L(\Delta L) + MP_K(\Delta K) = 0$$

$$\Rightarrow \frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L} = MRTS_{L,K}$$

Marginal Rate of Technical Substitution

- The rate at which the quantity of capital that can be *decreased* for every unit of *increase* in the quantity of labor, holding the quantity of output constant,

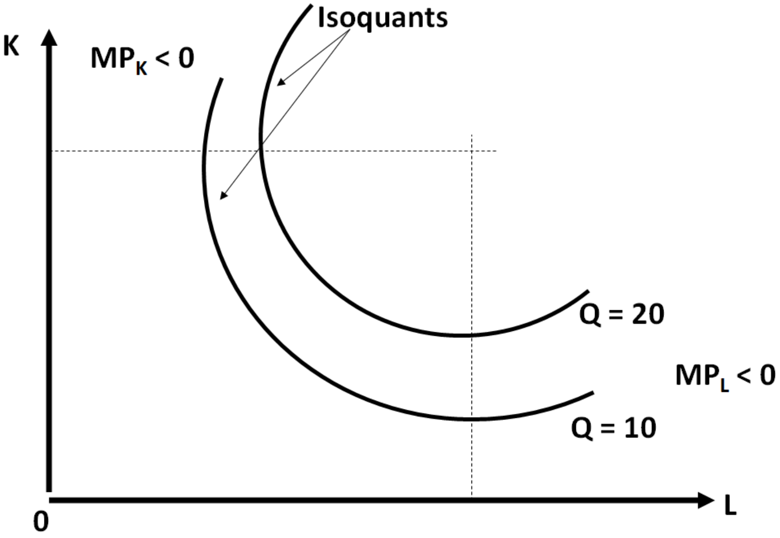
  or

- The rate at which the quantity of capital that can be *increased* for every unit of *decrease* in the quantity of labor, holding the quantity of output constant.

Marginal Rate of Technical Substitution

- If both marginal products are positive, the slope of the isoquant is negative.
- If we have diminishing marginal returns, we also have a diminishing marginal rate of technical substitution - the marginal rate of technical substitution of labor for capital diminishes as the quantity of labor increases, along an isoquant - isoquants are convex to the origin.
- For many production functions, marginal products eventually become negative. Why don't most graphs of Isoquants include the upwards-sloping portion?

Isoquants



Marginal Rate of Technical Substitution

$$\Delta Q = (\Delta K) MP_K + (\Delta L) MP_L$$

$$MP_L = \frac{\Delta Q}{\Delta L}\Big|_{K \text{ is held constant}} \qquad MP_K = \frac{\Delta Q}{\Delta K}\Big|_{L \text{ is held constant}}$$

$$\Rightarrow \frac{MP_L}{MP_K} = MRTS_{L,K}$$

Elasticity of Substitution

- A measure of how easy is it for a firm to substitute labor for capital.
- It is the percentage change in the capital-labor ratio for every one percent change in the $MRTS_{L,K}$ along an isoquant.

Elasticity of Substitution

Elasticity of Substitution

The **elasticity of substitution**, σ, measures how the capital-labor ratio, K/L, changes relative to the change in the $MRTS_{L,K}$.

$$\sigma = \frac{\text{Percentage change in capital-labor ratio}}{\text{Percentage change in } MRTS_{L,K}}$$

$$\sigma = \frac{\%\Delta(\frac{K}{L})}{\%\Delta MRTS_{L,K}}$$

Elasticity of Substitution

Example

Suppose that

$$MRST^{A}_{L,K} = 4, \ \frac{K^A}{L^A} = 4$$

$$MRST^{B}_{L,K} = 1, \ \frac{K^B}{L^B} = 1$$

Elasticity of Substitution

Example

Suppose that

$$MRST^A_{L,K} = 4,\ \frac{K^A}{L^A} = 4$$

$$MRST^B_{L,K} = 1,\ \frac{K^B}{L^B} = 1$$

$$\Delta MRST_{L,K} =$$

Elasticity of Substitution

Example

Suppose that

$$MRST^A_{L,K} = 4, \ \frac{K^A}{L^A} = 4$$

$$MRST^B_{L,K} = 1, \ \frac{K^B}{L^B} = 1$$

$$\Delta MRST_{L,K} = MRST^B_{L,K} - MRST^A_{L,K} = -3$$

Elasticity of Substitution

Example

Suppose that

$$MRST_{L,K}^{A} = 4,\ \frac{K^A}{L^A} = 4$$

$$MRST_{L,K}^{B} = 1,\ \frac{K^B}{L^B} = 1$$

$$\Delta MRST_{L,K} = MRST_{L,K}^{B} - MRST_{L,K}^{A} = -3$$

$$\sigma =$$

Elasticity of Substitution

Example

Suppose that

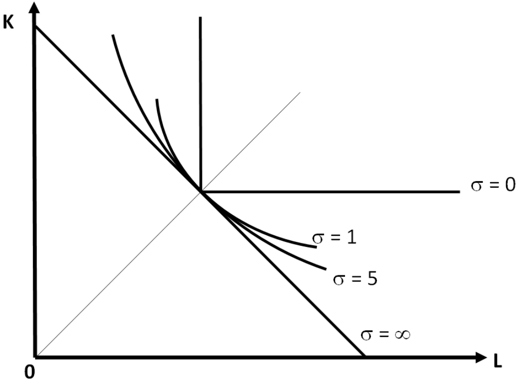
$$MRST^{A}_{L,K} = 4,\ \frac{K^A}{L^A} = 4$$

$$MRST^{B}_{L,K} = 1,\ \frac{K^B}{L^B} = 1$$

$$\Delta MRST_{L,K} = MRST^{B}_{L,K} - MRST^{A}_{L,K} = -3$$

$$\sigma = \frac{\Delta(\frac{K}{L})}{\Delta MRTS_{L,K}} \cdot \frac{MRTS_{L,K}}{\frac{K}{L}} = \frac{-3}{-3} \cdot \frac{4}{4} = 1$$

Elasticity of Substitution



The shape of the isoquant indicates the degree of substitutability of the inputs. . .

Returns to Scale

How much will output increase when ALL inputs increase by a particular amount?

$$\text{Return to Scale} = \frac{\%\Delta(\text{quantity of output})}{\%\Delta(\text{quantity of } \textit{all} \text{ inputs})}$$

Returns to Scale

Let λ represent the amount by which both inputs, labor and capital, increase.

$$Q = f(\lambda L, \lambda K) \text{ for } \lambda > 1$$

Let us ϕ represent the resulting proportionate increase in output, Q

- Increasing returns: $\phi > \lambda$
- Constant returns: $\phi = \lambda$
- Decreasing returns: $\phi > \lambda$

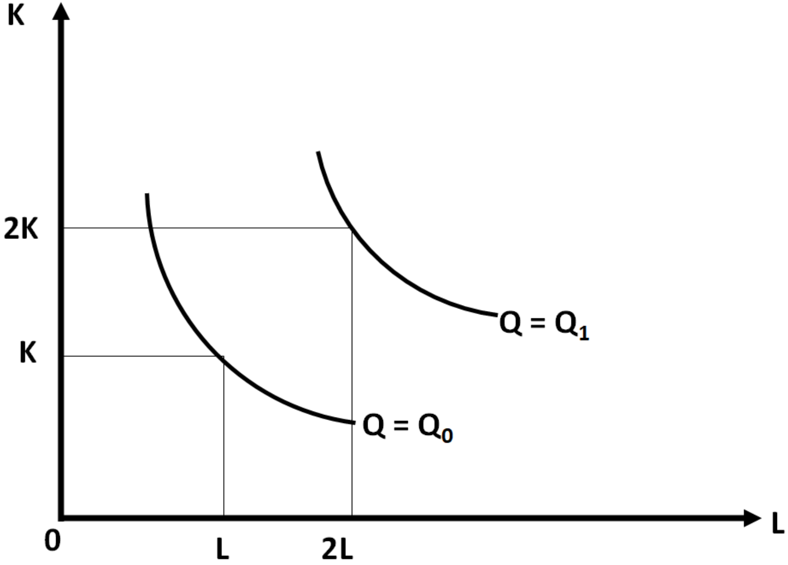
Returns to Scale

- How much will output increase when ALL inputs increase by a particular amount?

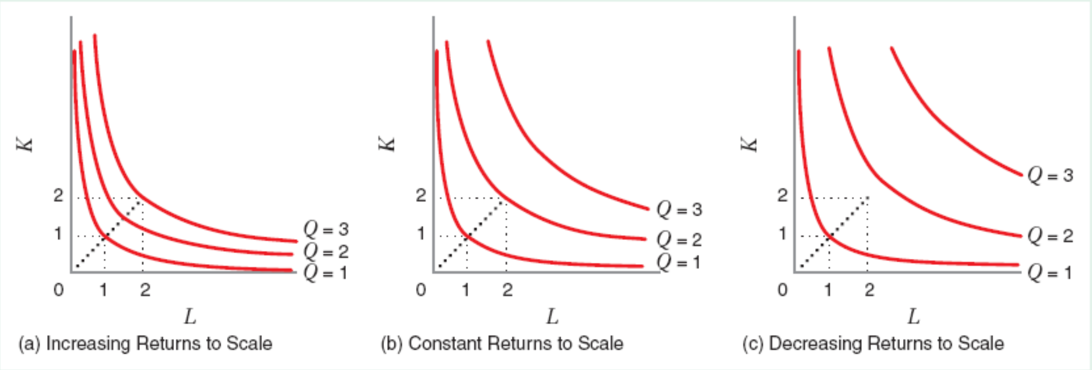
$$RTS = \frac{\%\Delta Q}{\%\Delta \text{ (all inputs)}}$$

- If a 1% increase in all inputs results in a greater than 1% increase in output, then the production function exhibits **increasing returns to scale**.
- If a 1% increase in all inputs results in exactly a 1% increase in output, then the production function exhibits **constant returns to scale**.
- If a 1% increase in all inputs results in a less than 1% increase in output, then the production function exhibits **decreasing returns to scale**.

Returns to Scale



Returns to Scale

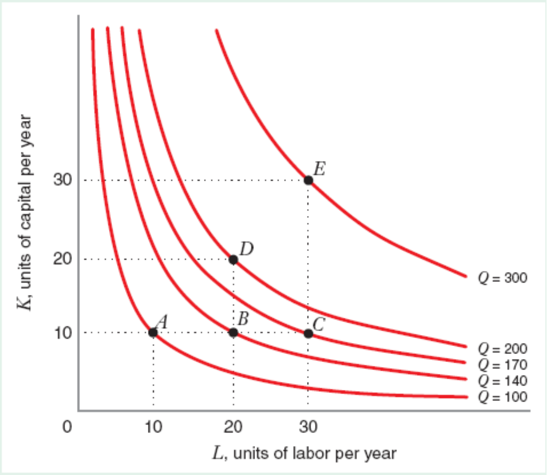


Returns to Scale vs. Marginal Return

- Returns to scale: all inputs are increased simultaneously.
- Marginal Returns: Increase in the quantity of a single input holding all others constant.

- The marginal product of a single factor may diminish while the returns to scale do not.
- Returns to scale need not be the same at different levels of production.

Returns to Scale vs. Marginal Return



Production function with *CRTS* but diminishing marginal returns to labor.

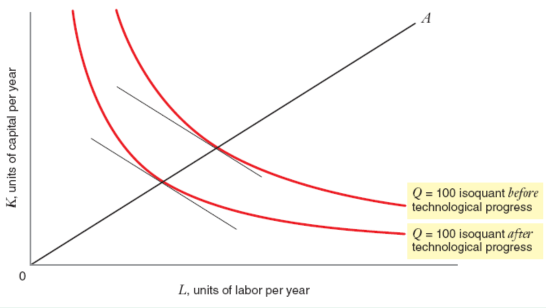
Technical Progress

Technical Progress

Technological progress (or **invention**) shifts the production function by allowing the firm to achieve more output from a given combination of inputs (or the same output with fewer inputs).

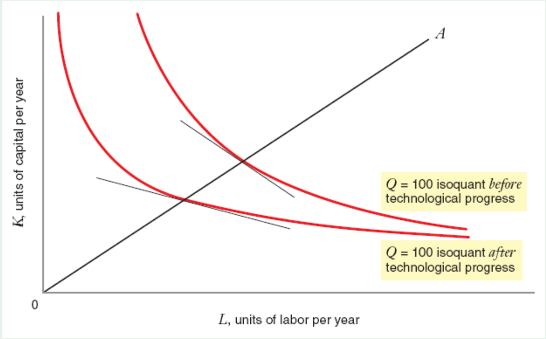
- **Labor saving technological progress** results in a fall in the $MRTS_{L,K}$ along any ray from the origin.
- **Capital saving technological progress** results in a rise in the $MRTS_{L,K}$ along any ray from the origin.

(Neutral) Technological Progress



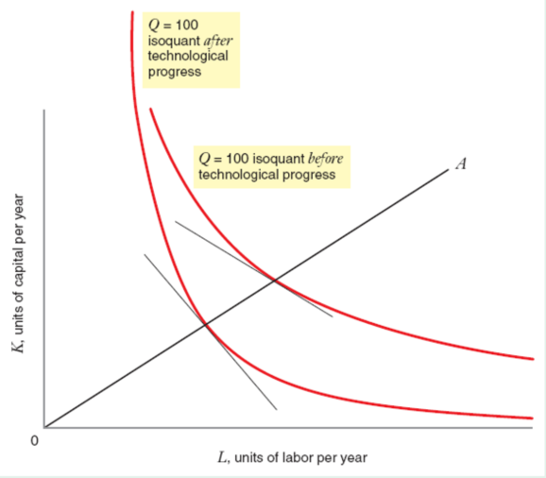
Technological progress that decreases the amounts of labor and capital needed to produce a given output. Affects $MRTS_{K,L}$.

Labor saving Technological Progress



Technological progress that causes the marginal product of capital to increase relative to the marginal product of labor.

Capital saving Technological Progress



Technological progress that causes the marginal product of labor to increase relative to the marginal product of capital.

Costs and Cost Minimization

Adapted from Chapter 7 of Besanko's Microeconomics

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Department of Economics
University of Innsbruck

May 14, 2025

Overview

Explicit Costs and Implicit Costs

Explicit Costs

Explicit Costs are costs that involve a direct monetary outlay.

Implicit Costs

Implicit Costs are costs that do not involve outlays of cash.

Opportunity Cost

Opportunity Cost

The relevant concept of cost is **opportunity cost**: the value of a resource in its best alternative use.
The only alternative we consider is the **best** alternative.

Economic Costs and Accounting Costs

Economic Costs

Economic Costs is the sum of a firm's explicit costs and implicit Costs.

Accounting Costs

Accounting Costs is the total of a firm's explicit costs.

Sunk Costs

Sunk Costs

Sunk Costs are costs that must be incurred no matter what the decision. These costs are not part of opportunity costs.

Example

Bowling Ball Factory

- It costs $\$5M$ to build and has no alternative uses
- $\$5M$ is not sunk cost for the decision of whether or not to build the factory
- $\$5M$ is sunk cost for the decision of whether to operate or shut down the factory

Non-Sunk Costs

Non-Sunk Costs are costs that must be incurred only if a particular decision is made.

Cost Minimization

Cost minimization problem

Cost minimization problem: Finding the input combination that minimizes a firm's total cost of producing a particular level of output.

Cost minimization firm

Cost minimization firm: A firm that seeks to minimize the cost of producing a given amount of output.

Long run

Long run: A period of time when the quantities of all of the firm's input can vary.

Short run

Short run: A period of time when at least one of its inputs' quantities is fixed.

Long Run Cost Minimization

Minimize the firm's costs, subject to a firm producing a given amount of output.
Cost to the Firm:

$$TC = wL + rK$$

- TC: total cost
- w: wage rate
- L: quantity of labor
- r: price per unit of capital services
- K: quantity of capital

Isocost Line

Isocost Line

The set of combinations of labor and capital that yield the same total cost for the firm.

Isocost Line

Example

Isocost Line Let us assume:

$$w = \$10$$

$$r = \$20$$

Isocost Line

Example

Isocost Line Let us assume:

$$w = \$10$$

$$r = \$20$$

What is the isocost line relative to $TC = \$1M$?

Isocost Line

Example

Isocost Line Let us assume:

$$w = \$10$$

$$r = \$20$$

What is the isocost line relative to $TC = \$1M$?

$$1,000,000 = 10L + 20K \Rightarrow K = 50,000 - \frac{1}{2}L$$

Isocost Line

Example

Isocost Line Let us assume:

$$w = \$10$$

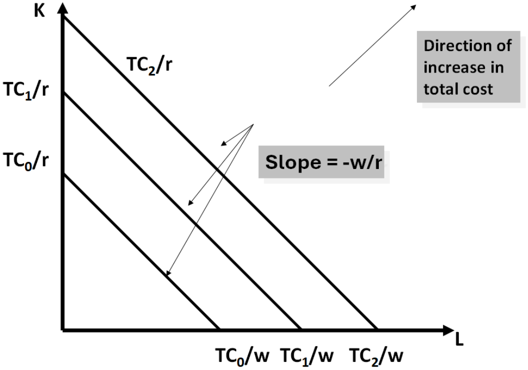
$$r = \$20$$

What is the isocost line relative to $TC = \$1M$?

$$1,000,000 = 10L + 20K \Rightarrow K = 50,000 - \frac{1}{2}L$$

Or more generally:

$$K = \frac{TC}{r} - \frac{w}{r}L$$



Combinations of labor and capital that yields the same total cost for the firm.

Long-Run Cost Minimization

Suppose that a firm's owners wish to minimize costs.
Let the desired output be Q_0
Technology: $Q = f(L, K)$

Long-Run Cost Minimization

Owner's problem

$$\min TC = rK + wL$$

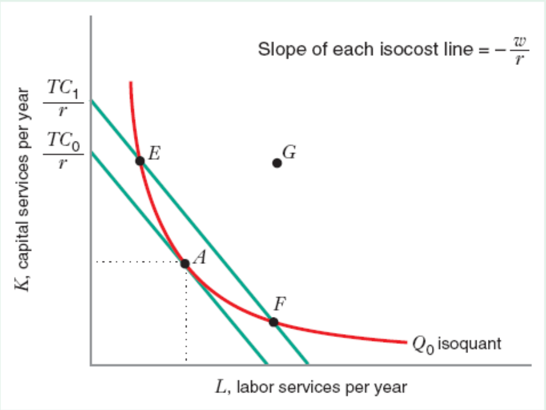
$$s.t.\ Q_0 = f(L, K)$$

Where $TC = rK + wL$ or $K = \frac{TC}{r} - \frac{w}{r}L$ is the isocost line.

Long-Run Cost Minimization

- Cost minimization subject to satisfaction of the isoquant equation: $Q_0 = f(L, K)$
- Note: analogous to expenditure minimization for the consumer
- **Tangency Condition**: $MRTS_{L,K} = -\frac{MP_L}{MP_K} = -\frac{w}{r}$ or $\frac{MP_L}{w} = \frac{MP_K}{r}$
- Constrain: $Q_0 = f(L, K)$

Long-Run Cost Minimization



Solution to cost minimization:
Point where isoquant is just **tangent** to isocost line (A).
G – Technically Inefficient
E & F – Technically Efficient but do not minimize cost

Long-Run Cost Minimization

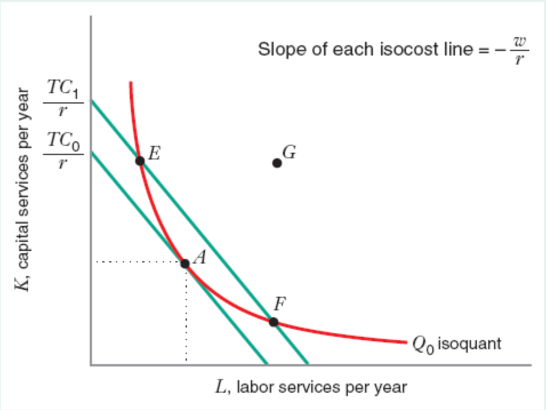
Solution to cost minimization:
Slope of isoquant = slope of isocost line

$$MRTS_{L,K} = \frac{w}{r} \text{ or } \frac{MP_L}{MP_K} = \frac{w}{r}$$

Ratio of marginal products = ratio of input prices

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

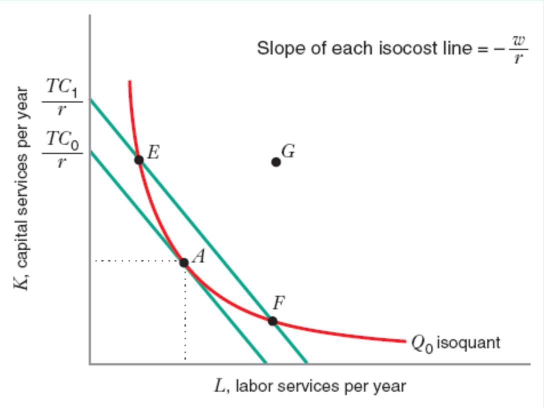
Long-Run Cost Minimization



At point E: $\frac{MP_L}{MP_K} > \frac{w}{r}$ or $\frac{MP_L}{w} > \frac{MP_K}{r}$

This implies the firm could spend an additional dollar on labor and save more than a dollar by reducing its employment of capital and keep output constant.

Long-Run Cost Minimization



At point F: $\frac{MP_L}{MP_K} < \frac{w}{r}$ or $\frac{MP_L}{w} < \frac{MP_K}{r}$

This implies the firm could spend an additional dollar on capital and save more than a dollar by reducing its employment of labor and keep output constant.

Interior Solution

Example

Assume: $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$

Interior Solution

Example

Assume: $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$

$MP_L =$

Interior Solution

Example

Assume: $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$

$$MP_L = 25L^{-\frac{1}{2}}K^{\frac{1}{2}},$$

Interior Solution

Example

Assume: $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$

$$MP_L = 25L^{-\frac{1}{2}}K^{\frac{1}{2}}, \; MP_K =$$

Interior Solution

Example

$$\text{Assume: } Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$$

$$MP_L = 25L^{-\frac{1}{2}}K^{\frac{1}{2}}, \; MP_K = 25L^{\frac{1}{2}}K^{-\frac{1}{2}}$$

Interior Solution

Example

Assume: $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$

$$MP_L = 25L^{-\frac{1}{2}}K^{\frac{1}{2}},\ MP_K = 25L^{\frac{1}{2}}K^{-\frac{1}{2}}$$

$$w = \$5,\ r = \$20,\ Q_0 = 1000$$

Interior Solution

Example

$$\text{Assume: } Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$$

$$MP_L = 25L^{-\frac{1}{2}}K^{\frac{1}{2}},\ MP_K = 25L^{\frac{1}{2}}K^{-\frac{1}{2}}$$

$$w = \$5,\ r = \$20,\ Q_0 = 1000$$

$$\frac{MP_L}{MP_K} = \frac{K}{L} \Rightarrow \frac{K}{L} = \frac{5}{20} \Rightarrow L = 4K$$

$$1000 = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$$

Interior Solution

Example

$$\text{Assume: } Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$$

$$MP_L = 25L^{-\frac{1}{2}}K^{\frac{1}{2}},\ MP_K = 25L^{\frac{1}{2}}K^{-\frac{1}{2}}$$

$$w = \$5,\ r = \$20,\ Q_0 = 1000$$

$$\frac{MP_L}{MP_K} = \frac{K}{L} \Rightarrow \frac{K}{L} = \frac{5}{20} \Rightarrow L = 4K$$

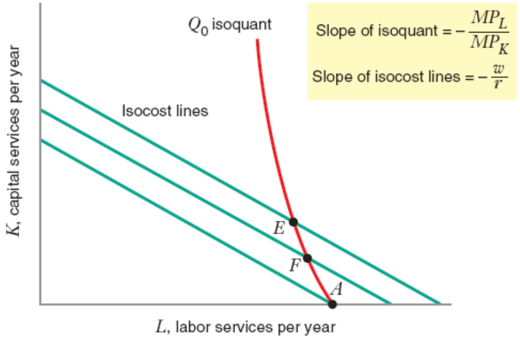
$$1000 = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$$

$$K^* = 10,\ L^* = 40$$

Corner Solution

The cost-minimizing input combination for producing Q_0 units of output occurs at point A where the firms uses no capital. At this corner point the isocost line is flatter than the isoquant.

$$\frac{MP_L}{MP_K} > \frac{w}{r} \Rightarrow \frac{MP_L}{w} > \frac{MP_K}{r}$$



Corner Solution

Example

Assume: $Q = 10L + 2K$

Corner Solution

Example

Assume: $Q = 10L + 2K$

$$MP_L =$$

Corner Solution

Example

Assume: $Q = 10L + 2K$

$MP_L = 10,$

Corner Solution

Example

Assume: $Q = 10L + 2K$

$$MP_L = 10, \; MP_K =$$

Corner Solution

Example

Assume: $Q = 10L + 2K$

$$MP_L = 10, \; MP_K = 2$$

Corner Solution

Example

Assume: $Q = 10L + 2K$

$$MP_L = 10,\ MP_K = 2$$

$$w = \$5,\ r = \$2,\ Q_0 = 200$$

Corner Solution

Example

$$\text{Assume: } Q = 10L + 2K$$

$$MP_L = 10, \; MP_K = 2$$

$$w = \$5, \; r = \$2, \; Q_0 = 200$$

$$\frac{MP_L}{MP_K} = \frac{10}{2} > \frac{w}{r} = \frac{5}{2}$$

Corner Solution

Example

$$\text{Assume: } Q = 10L + 2K$$

$$MP_L = 10, \; MP_K = 2$$

$$w = \$5, \; r = \$2, \; Q_0 = 200$$

$$\frac{MP_L}{MP_K} = \frac{10}{2} > \frac{w}{r} = \frac{5}{2}$$

But the *bang for the buck* in labor larger than the *bang for the buck* in capital

Corner Solution

Example

$$\text{Assume: } Q = 10L + 2K$$

$$MP_L = 10, \ MP_K = 2$$

$$w = \$5, \ r = \$2, \ Q_0 = 200$$

$$\frac{MP_L}{MP_K} = \frac{10}{2} > \frac{w}{r} = \frac{5}{2}$$

But the *bang for the buck* in labor larger than the *bang for the buck* in capital

$$\frac{MP_L}{w} = \frac{10}{5} > \frac{MP_K}{r} = \frac{2}{2}$$

Corner Solution

Example

$$\text{Assume: } Q = 10L + 2K$$

$$MP_L = 10, \; MP_K = 2$$

$$w = \$5, \; r = \$2, \; Q_0 = 200$$

$$\frac{MP_L}{MP_K} = \frac{10}{2} > \frac{w}{r} = \frac{5}{2}$$

But the *bang for the buck* in labor larger than the *bang for the buck* in capital

$$\frac{MP_L}{w} = \frac{10}{5} > \frac{MP_K}{r} = \frac{2}{2}$$

$$K^* = 0, \; L^* = 20$$

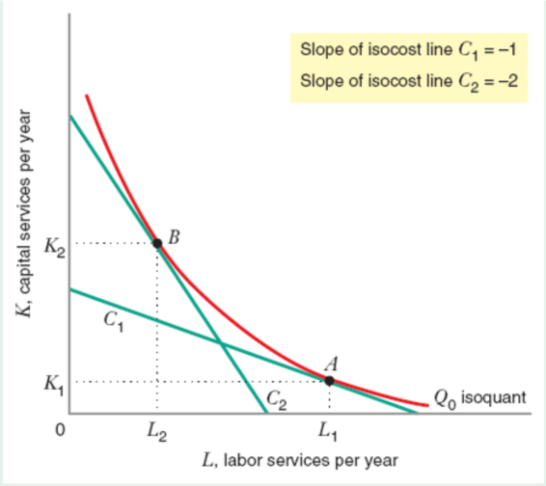
Comparative Statics

A change in the relative price of inputs changes the slope of the isocost line.

All else equal, an increase in w must decrease the cost minimizing quantity of labor and increase the cost minimizing quantity of capital with diminishing $MRTS_{L,K}$.

All else equal, an increase in r must decrease the cost minimizing quantity of capital and increase the cost minimizing quantity of labor.

Change in Reltive Prices of Inputs



- Price of capital $r = 1$.
- Quantity of output Q_0 is constant.
- When price of labor $w = 1$ the isocost line is C_1, optimal point A.
- When price of labor $w = 2$ isocost line is C_2, optimal point B.

Some Key Definitions

An increase in Q_0 moves the isoquant Northeast.

Expansion Path

Expansion Path: A line that connects the cost-minimizing input combinations as the quantity of output, Q, varies, holding input prices constant.

Normal Inputs

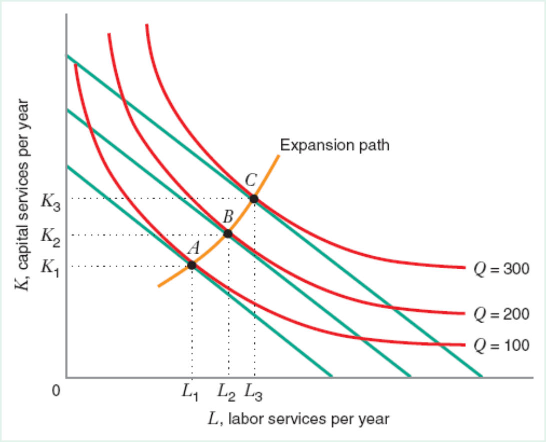
Normal Inputs: An input whose cost-minimizing quantity increases as the firm produces more output.

Inferior Input

Inferior Input: An input whose cost-minimizing quantity decreases as the firm produces more output.

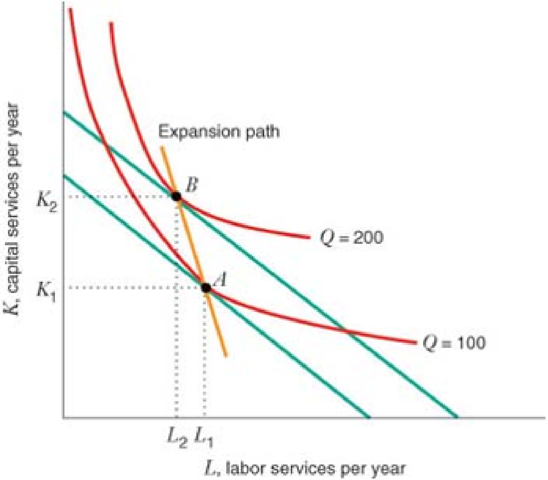
An Expansion Path

As output increases, the cost minimization path moves from point A to B to C when inputs are normal.



An Expansion Path

As output increases, the cost minimization path moves from point A to B to C when labor is an inferior input.



Inputs Demand

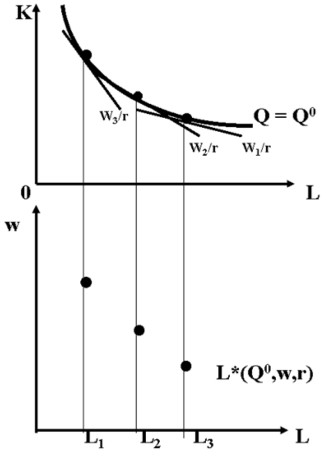
Inputs Demand

A function that shows how the firm's cost-minimizing quantity of input varies with the price of that input.

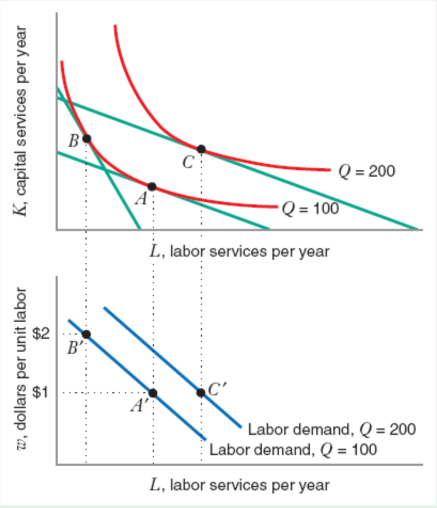
Labor demand curve: Shows how the firm's cost-minimizing quantity of labor varies with the price of labor.

Capital demand curve: Shows how the firm's cost-minimizing quantity of capital varies with the price of capital.

Input Demand Functions



Input Demand



For a fixed quantity, as price of labor increases from \$1 to \$2, firm moves along its labor demand curve from A to B. Increase in output shifts the demand curve.

Price Elasticity of Demand for Inputs

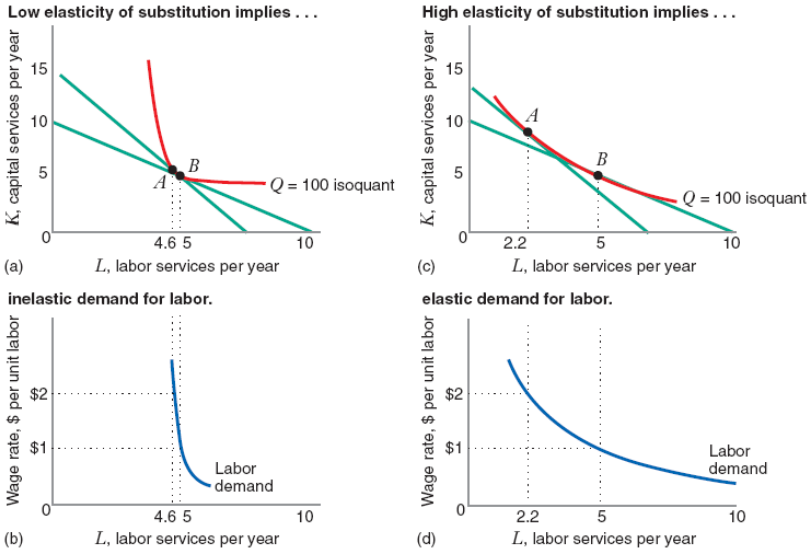
Percentage change in the cost-minimizing quantity of labor with respect to a 1% change in the price of labor.

$$\varepsilon_{L,w} = \frac{\Delta L}{\Delta w}\frac{w}{L}$$

Percentage change in the cost-minimizing quantity of capital with respect to a 1% change in the price of capital.

$$\varepsilon_{K,r} = \frac{\Delta K}{\Delta r}\frac{r}{K}$$

Price Elasticity of Demand for Inputs



Short-Run Cost Minimization

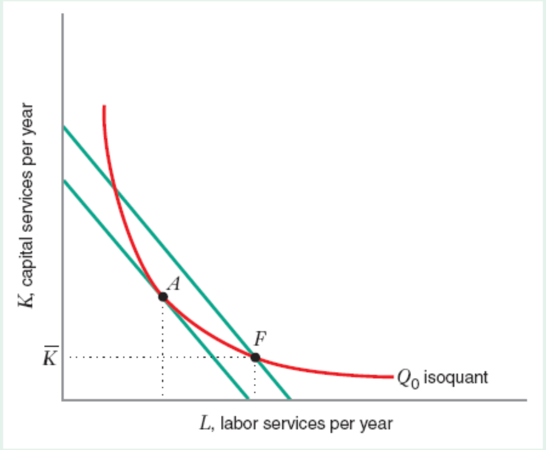
Total Variable Costs

Total Variable Costs – the sum of total expenditures on variable inputs, such as labor and materials, at the short-run cost-minimizing input combination.

Total Fixed Costs

Total Fixed Costs – the cost of fixed inputs; it does not vary with output

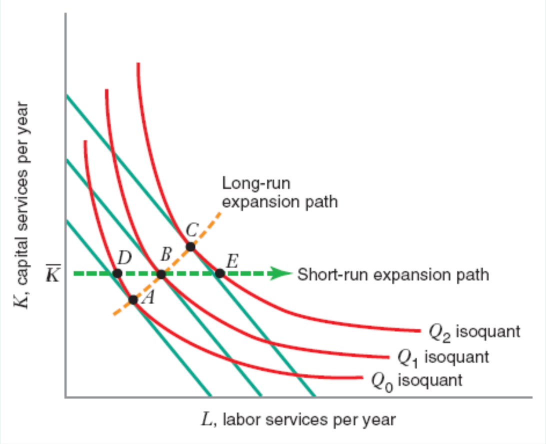
Short-Run Cost Minimization



One fixed Input - Capital $\bar{K}$

- Short run combination is point F
- If the firm were free to adjust all of its inputs, the cost-minimizing combination is at Point A

Short-Run Cost Minimization



- Long run-all variables are variable and the expansion path is from $A - B - C$
- Short run-some variables are fixed (capital)-the expansion path is from D $-E - F$

Short-Run Cost Minimization

- Short run: One input is fixed, capital. Firm can vary the other input, labor. So demand for labor will be independent of price.
- Short run demand for labor will also depend on quantity produced. As quantity increased, labor used increases holding capital fixed.

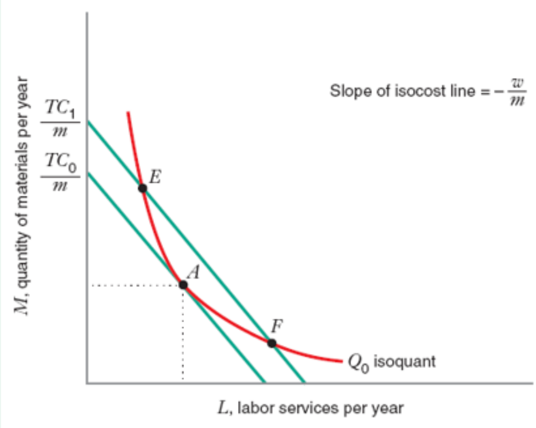
Short-Run Cost Minimization

$$Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}} = 1000$$

Capital is fixed $\bar{K}$

$$L = \frac{Q^2}{2500\bar{K}}$$

Short-Run Cost Minimization



- More than one variable input – analysis similar to long-run cost minimization
- 3 inputs – labor (L), capital ($\bar{K}$), raw materials (M)

$$MRTS_{L,M} = \frac{w}{m}$$

$$\frac{MP_L}{MP_M} = \frac{w}{m}$$

Cost Curves

Adapted from Chapter 8 of Besanko's Microeconomics

Luisa Lorè
Department of Economics
University of Innsbruck

May 16, 2025

Overview

Long Run Cost Functions

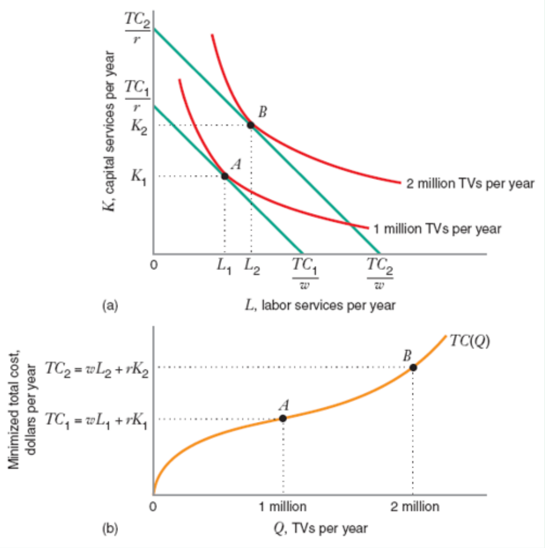
Long Run Cost Functions

The **long run total cost function** relates minimized total cost to output, Q, and to the factor prices (w and r).

$$TC(Q, w, r) = wL^*(Q, w, r) + rK^*(Q, w, r)$$

Where: L^* and K^* are the long run input demand functions

Long Run Cost Functions



As Quantity of output increases from 1 million to 2 million, with input prices(w, r) constant, cost minimizing input combination moves from TC_1 to TC_2 which gives the $TC(Q)$ curve.

Long Run Cost Functions

Example

What is the long run total cost function for production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$?

Long Run Cost Functions

Example

What is the long run total cost function for production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$?

$$L^*(Q, w, r) = \frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}}$$

$$K^*(Q, w, r) = \frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

Long Run Cost Functions

Example

What is the long run total cost function for production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$?

$$L^*(Q, w, r) = \frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}}$$

$$K^*(Q, w, r) = \frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

$$TC(Q, w, r) = w\left[\frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}}\right] + r\left[\frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}\right]$$

$$= \left(\frac{Q}{50}\right)(wr)^{\frac{1}{2}} + \left(\frac{Q}{50}\right)(wr)^{\frac{1}{2}} = \left(\frac{Q}{25}\right)(wr)^{\frac{1}{2}}$$

Long Run Cost Functions

Example

$$TC(Q, w, r) = \left(\frac{Q}{25}\right)(wr)^{\frac{1}{2}}$$

What is the graph of the total cost curve when $w = 25$ and $r = 100$?

Long Run Cost Functions

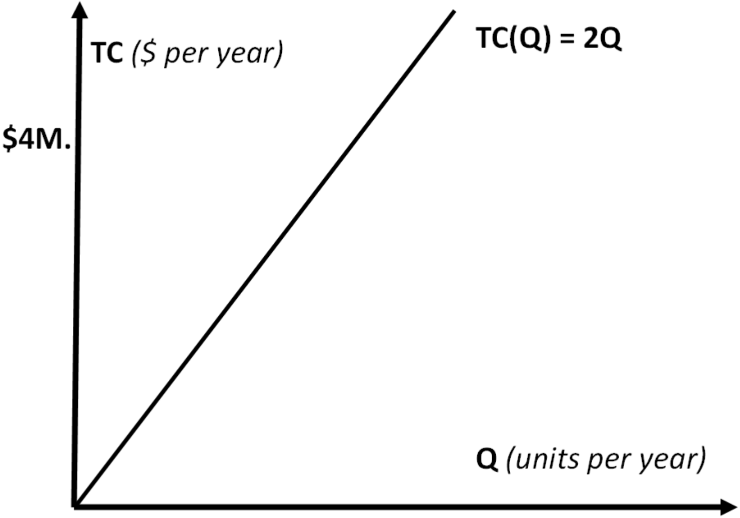
Example

$$TC(Q, w, r) = \left(\frac{Q}{25}\right)(wr)^{\frac{1}{2}}$$

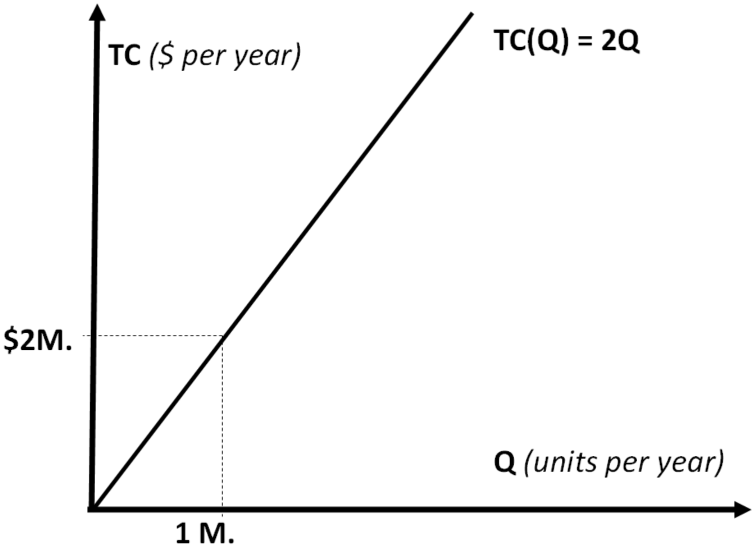
What is the graph of the total cost curve when $w = 25$ and $r = 100$?

$$TC(Q) = 2Q$$

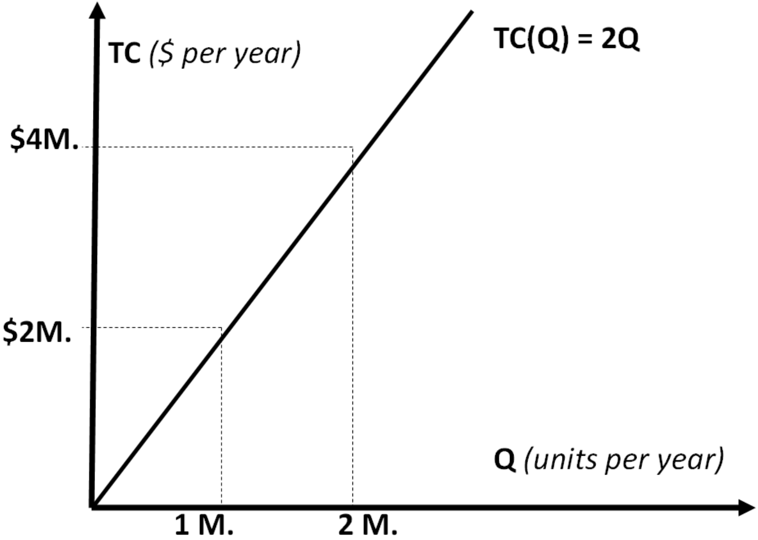
A Total Cost Curve



A Total Cost Curve



A Total Cost Curve



Long Run Total Cost Curve - Tracking Movement

Long Run Total Cost Curve

The **long run total cost curve** shows minimized total cost as output varies, holding input prices constant.

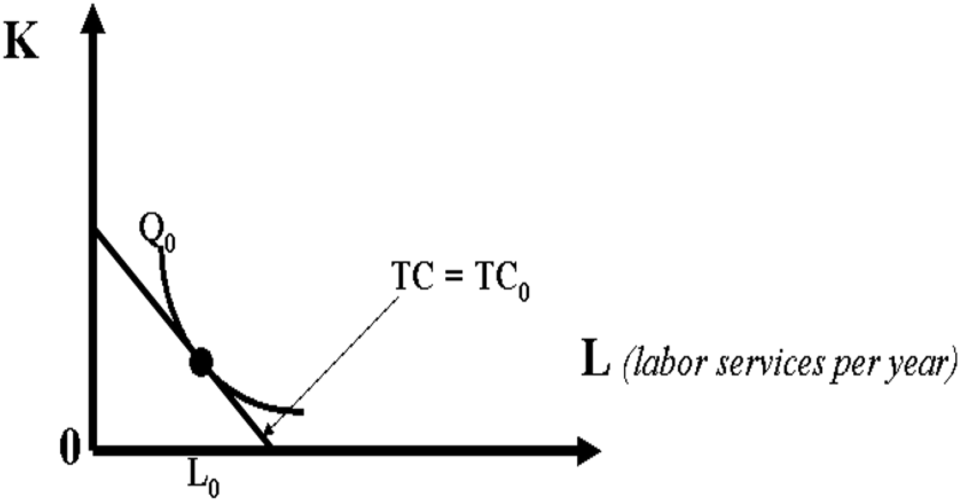
Long Run Total Cost Curve - Tracking Movement

Long Run Total Cost Curve

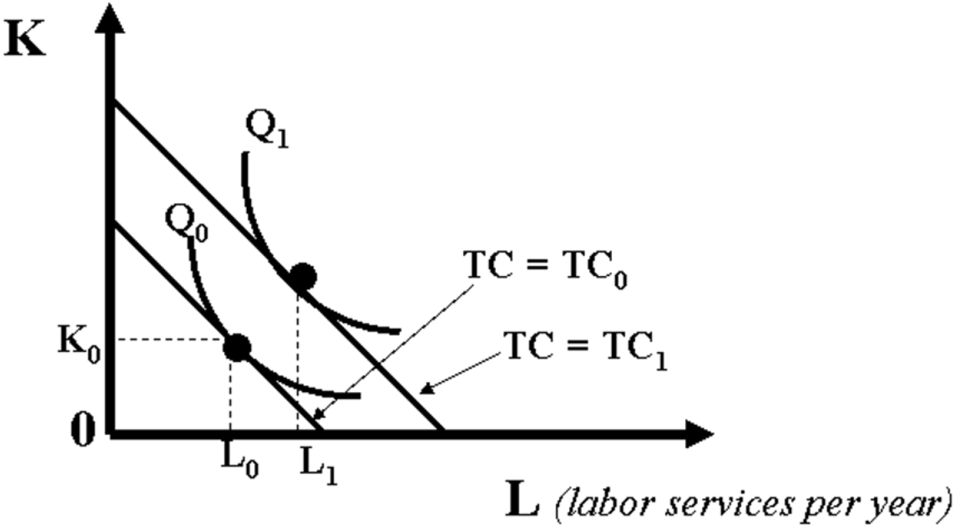
The **long run total cost curve** shows minimized total cost as output varies, holding input prices constant.

Graphically, what does the total cost curve look like if Q varies and w and r are fixed?

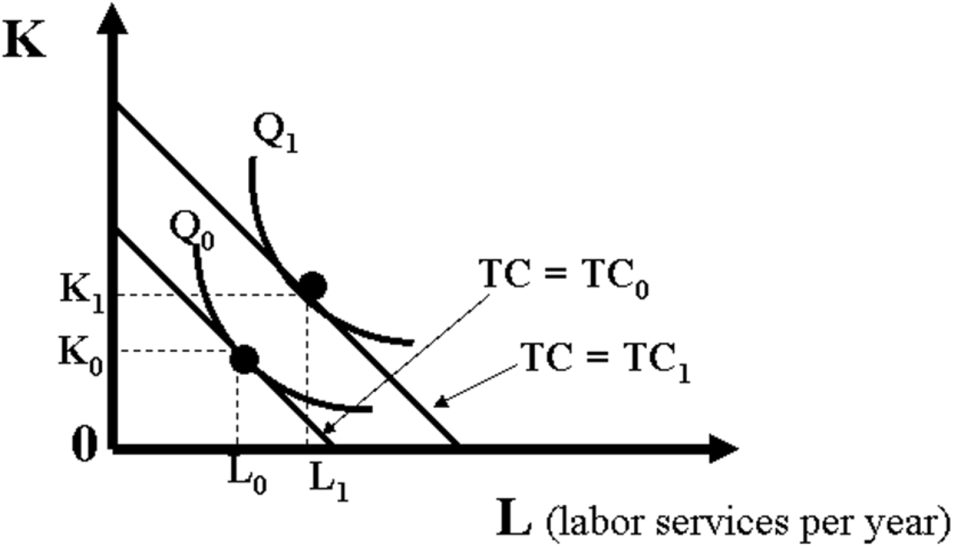
Long Run Total Cost Curve



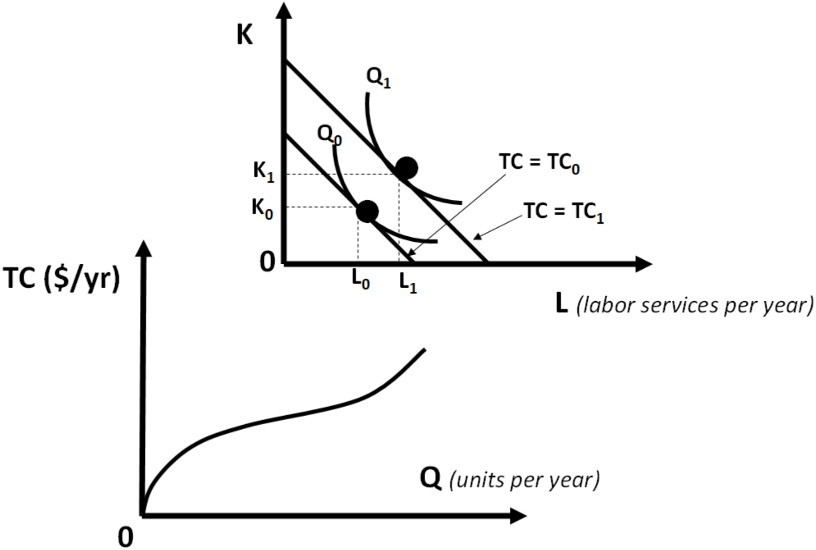
Long Run Total Cost Curve



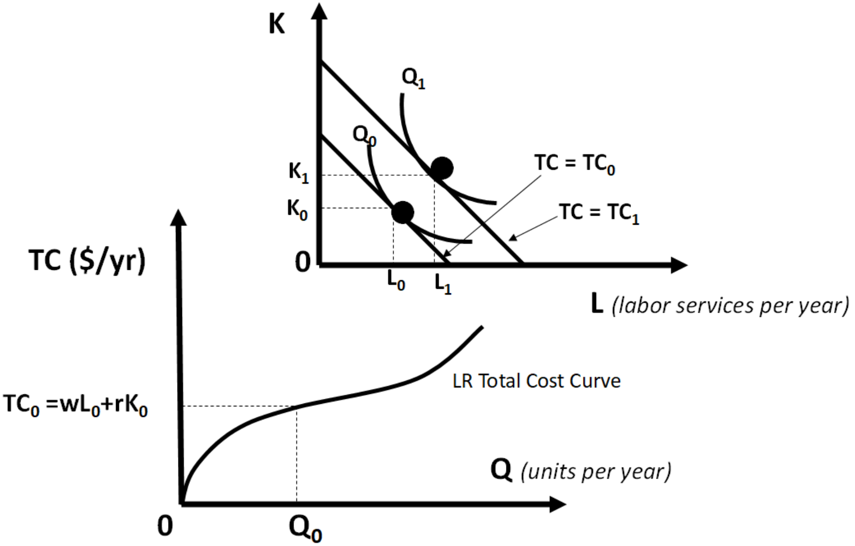
Long Run Total Cost Curve



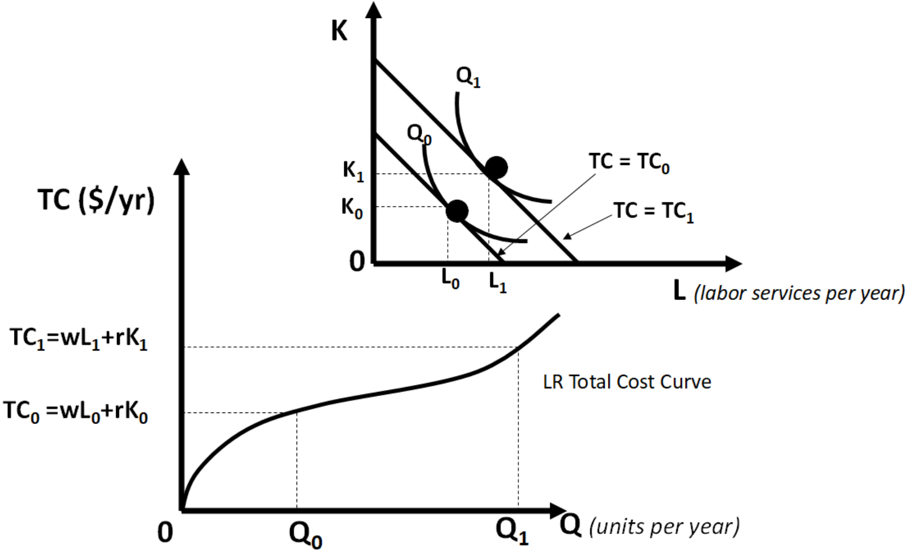
Long Run Total Cost Curve



Long Run Total Cost Curve



Long Run Total Cost Curve

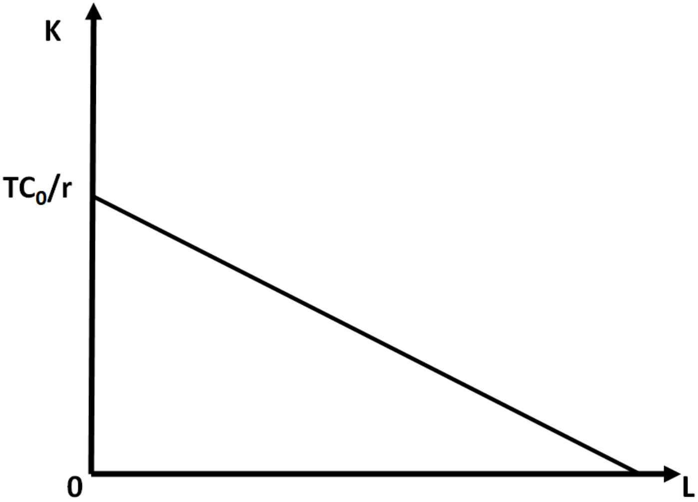


Long Run Total Cost Curve

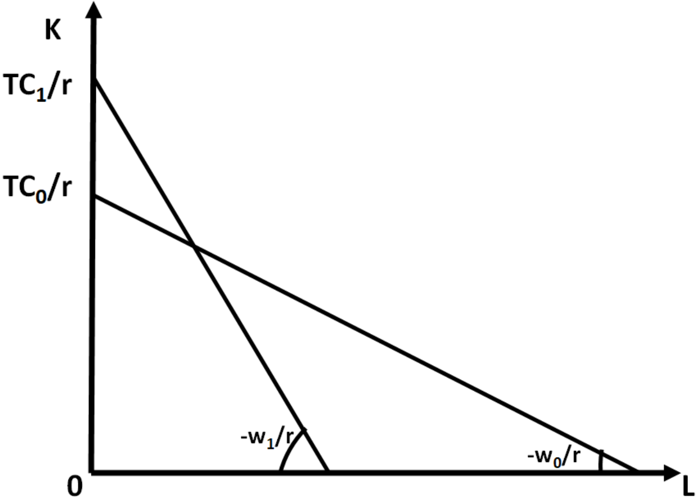
Identifying Shifts

Graphically, how does the total cost curve shift if wages rise but the price of capital remains fixed?

A Change in Input Prices



A Change in Input Prices



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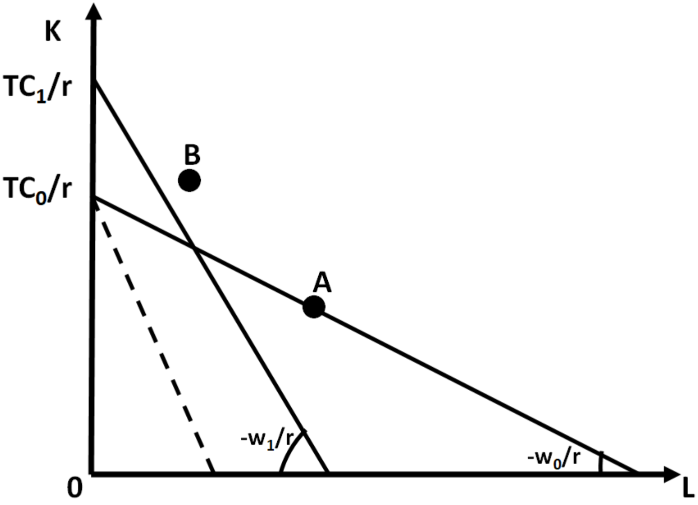

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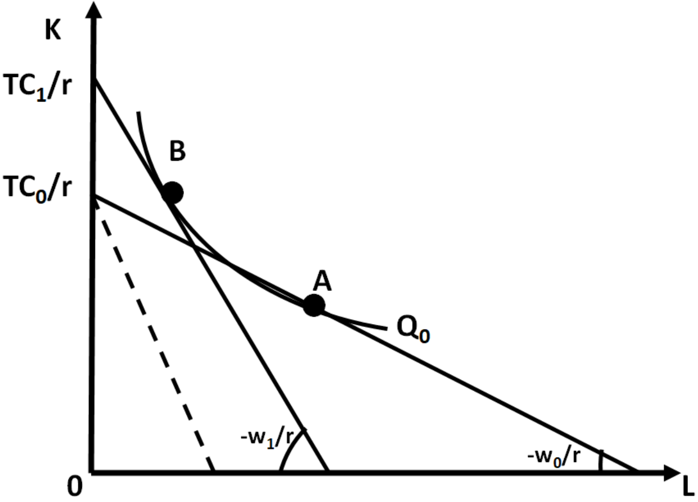
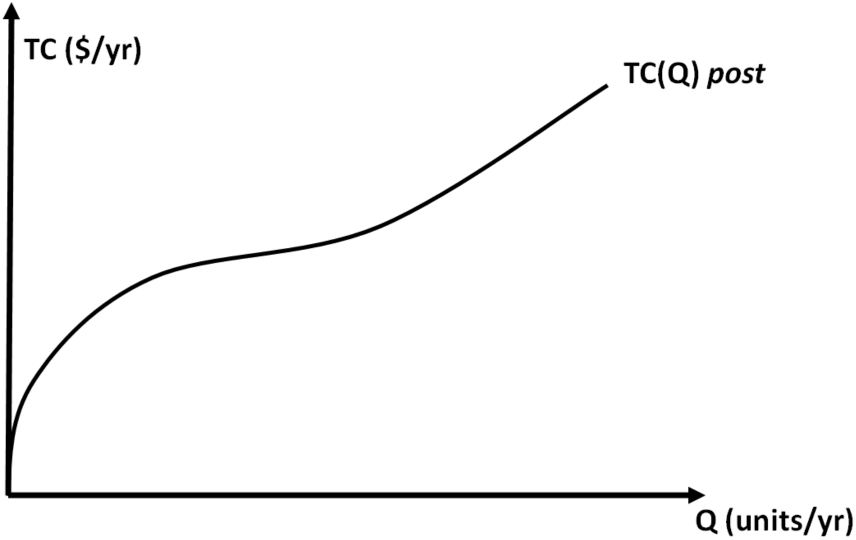


A Shift in the Total Cost Curve



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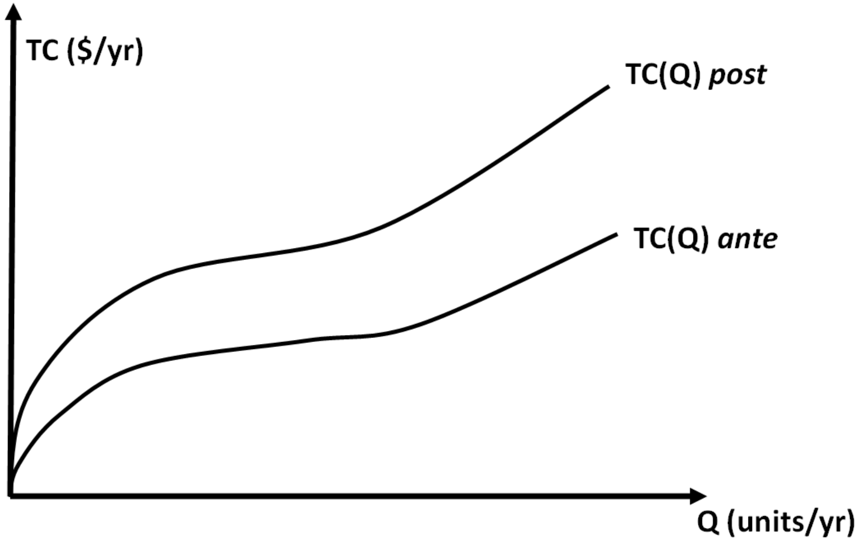
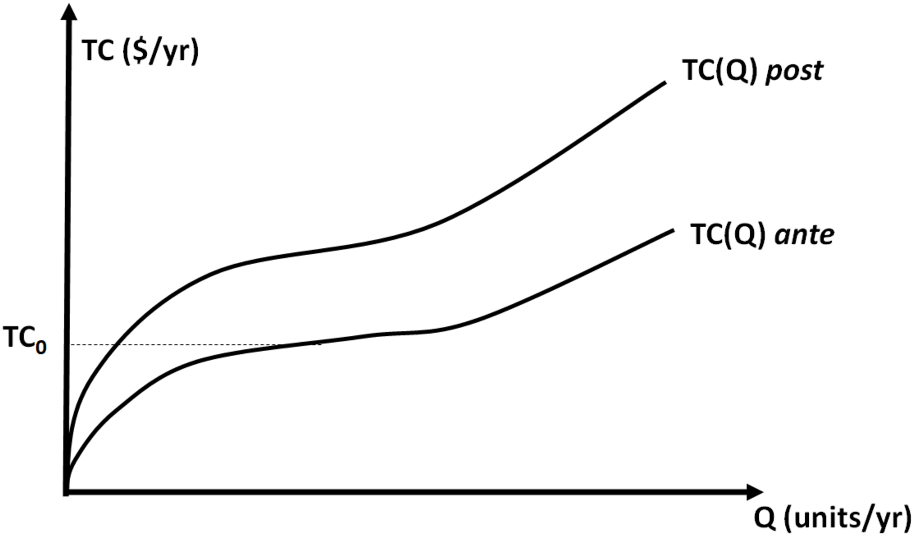
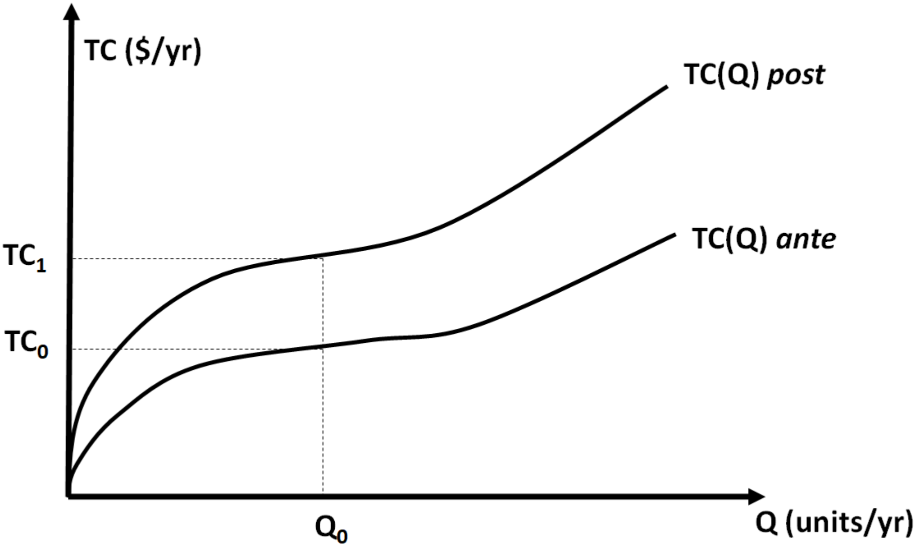


A Shift in the Total Cost Curve



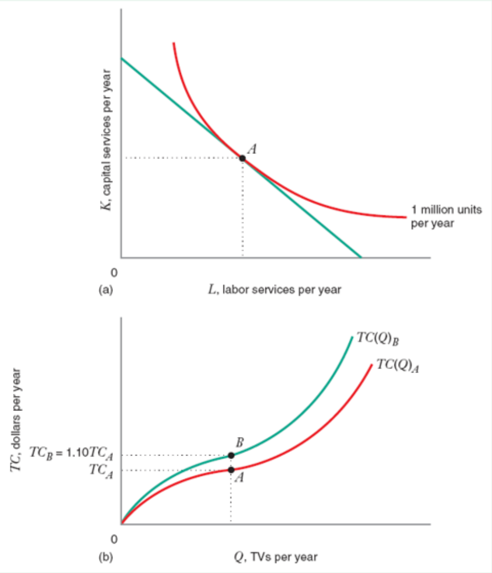
A Shift in the Total Cost Curve



Input Price Changes

How does the total cost curve shift if all input prices rise *(the same amount)*?

All Input Price Changes



Price of input increases proportionately by 10%. Cost minimization input stays same, slope of isoquant is unchanged. *TC* curve shifts up by the same 10 percent.

Long Run Average Cost Function

Long Run Average Cost Function

The **long run average cost function** is the long run total cost function divided by output, Q.

That is, the $LRAC$ function tells us the firm's cost per unit of output...

$$AC(Q, w, r) = \frac{TC(Q, w, r)}{Q}$$

Long Run Marginal Cost Function

The **long run marginal cost function** measures the rate of change of total cost as output varies, holding constant input prices.

$$MC(Q, w, r) = \frac{\Delta TC(Q, w, r)}{\Delta Q}$$

where w and r are constant.

Long Run Marginal Cost Function

Example

Recall that, for the production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$, the total cost function was $TC(Q, w, r) = \left(\frac{Q}{25}\right)(wr)^{\frac{1}{2}}$. If $w = 25$, and $r = 100$, $TC(Q) = 2Q$.

Long Run Marginal Cost Function

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a. What are the long run average and marginal cost functions for this production function?

Long Run Marginal Cost Function

Example

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a. What are the long run average and marginal cost functions for this production function?

$AC(Q, w, r) = \frac{(wr)^{\frac{1}{2}}}{25}$

$MC(Q, w, r) = \frac{(wr)^{\frac{1}{2}}}{25}$

Long Run Marginal Cost Function

Example

Recall that, for the production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$, the total cost function was $TC(Q, w, r) = \left(\frac{Q}{25}\right)(wr)^{\frac{1}{2}}$. If $w = 25$, and $r = 100$, $TC(Q) = 2Q$.

a. What are the long run average and marginal cost functions for this production function?

   $AC(Q, w, r) = \frac{(wr)^{\frac{1}{2}}}{25}$

   $MC(Q, w, r) = \frac{(wr)^{\frac{1}{2}}}{25}$

b. . What are the long run average and marginal cost curves when $w = 25$ and $r = 100$?

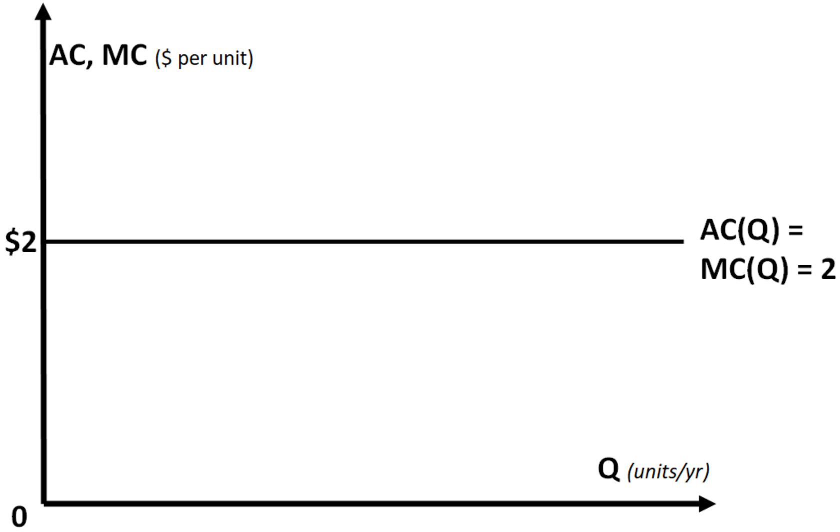
Long Run Marginal Cost Function

Example

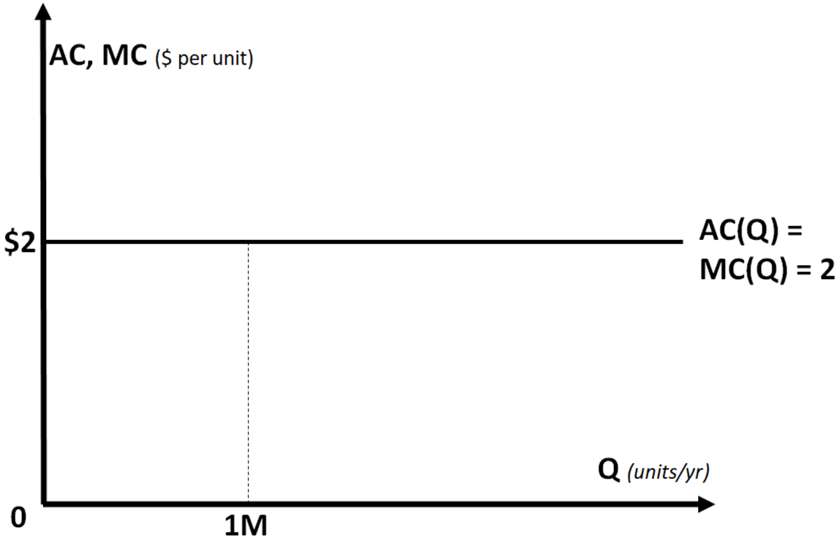
Recall that, for the production function $Q = 50L^{\frac{1}{2}}K^{\frac{1}{2}}$, the total cost function was $TC(Q, w, r) = \left(\frac{Q}{25}\right)(wr)^{\frac{1}{2}}$. If $w = 25$, and $r = 100$, $TC(Q) = 2Q$.

a. What are the long run average and marginal cost functions for this production function?
   $AC(Q, w, r) = \frac{(wr)^{\frac{1}{2}}}{25}$
   $MC(Q, w, r) = \frac{(wr)^{\frac{1}{2}}}{25}$
b. . What are the long run average and marginal cost curves when $w = 25$ and $r = 100$?
   $AC(Q) = \frac{2Q}{Q} = 2$
   $MC(Q) = \frac{\Delta(2Q)}{\Delta Q} = 2$

Average & Marginal Cost Curves



Average & Marginal Cost Curves



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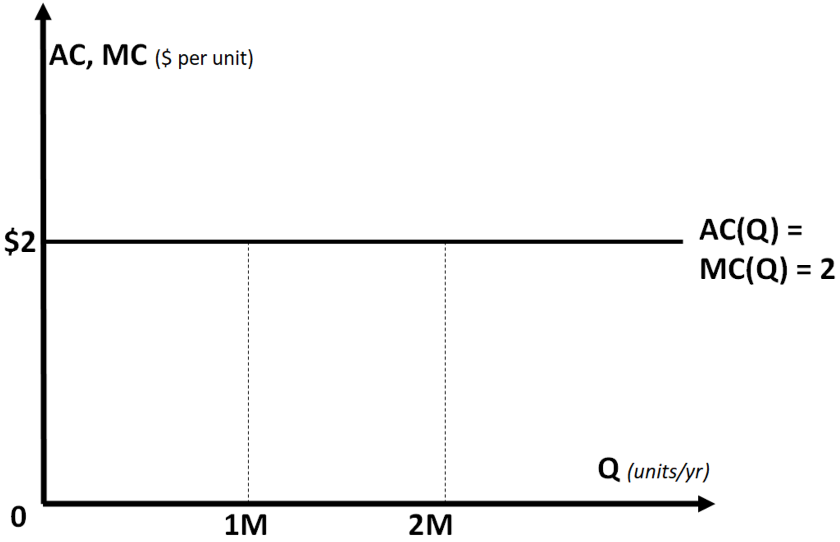


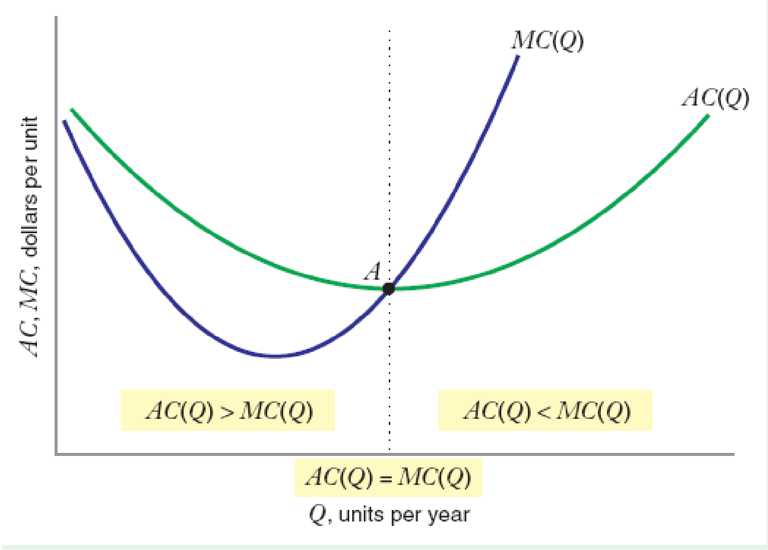
Average & Marginal Cost Curves

What is their relationship?

Suppose that w and r are fixed:

- When marginal cost is *less than* average cost, average cost is *decreasing in quantity*. That is, if $MC(Q) < AC(Q)$, $AC(Q)$ decreases in Q.
- When marginal cost is *greater than* average cost, average cost is *increasing in quantity*. That is, if $MC(Q) > AC(Q)$, $AC(Q)$ increases in Q.
- When marginal cost *equals* average cost, average cost *does not change with quantity*. That is, if $MC(Q) = AC(Q)$, $AC(Q)$ is flat with respect to Q.

Average & Marginal Cost Curves



Economies & Diseconomies of Scale

Economies of Scale

If average cost decreases as output rises, all else equal, the cost function exhibits **economies of scale**.

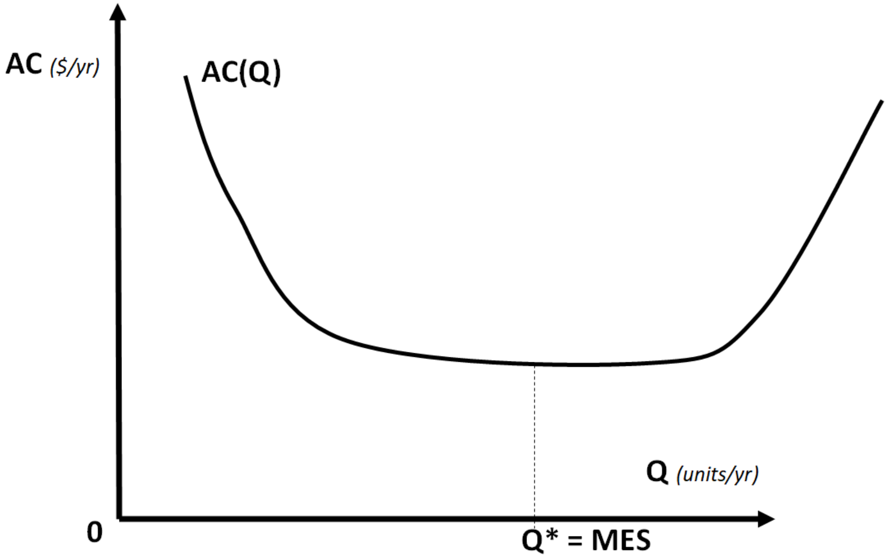
Diseconomies of Scale

If the average cost increases as output rises, all else equal, the cost function exhibits **diseconomies of scale**.

Minimum Efficient Scale

The smallest quantity at which the long run average cost curve attains its minimum point is called the **minimum efficient scale**.

Minimum Efficiency Scale (MES)



Returns to Scale & Economies of Scale

What is their relationship?

- When the production function exhibits *increasing returns to scale*, the long run average cost function exhibits *economies of scale* so that $AC(Q)$ decreases with Q, all else equal.
- When the production function exhibits *decreasing returns to scale*, the long run average cost function exhibits *diseconomies of scale* so that $AC(Q)$ increases with Q, all else equal.
- When the production function exhibits *constant returns to scale*, the long run average cost is flat: it neither increases nor decreases with output.

Output Elasticity of Total Cost

Output Elasticity of Total Cost

The percentage change in total cost per one percent change in output is the **output elasticity of total cost**, $\varepsilon_{TC,Q}$.

$$\varepsilon_{TC,Q} = \frac{\left(\frac{\Delta TC}{TC}\right)}{\left(\frac{\Delta Q}{Q}\right)} = \frac{\left(\frac{\Delta TC}{\Delta Q}\right)}{\left(\frac{TC}{Q}\right)} = \frac{MC}{AC}$$

- If $\varepsilon_{TC,Q} < 1$, $MC < AC$, so AC must be decreasing in Q. Therefore we have *economies of scale*.
- If $\varepsilon_{TC,Q} > 1$, $MC > AC$, so AC must be increasing in Q. Therefore we have *diseconomies of scale*.
- If $\varepsilon_{TC,Q} = 1$, $MC = AC$, so AC is just flat with respect to Q.

Short Run & Total Variable Cost Functions

Short Run Cost Function

The **short run total cost function** tells us the minimized total cost of producing Q units of output, when (at least) one input is fixed at a particular level.

Total Variable Cost Function

The **total variable cost function** is the minimized sum of expenditures on variable inputs at the short run cost minimizing input combinations.

Total Fixed Cost Function

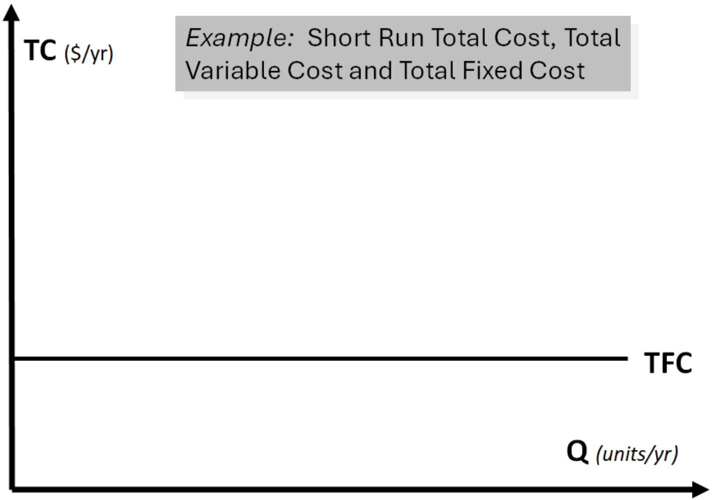
Total Fixed Cost Function

The total fixed cost function is a constant equal to the cost of the fixed input(s).

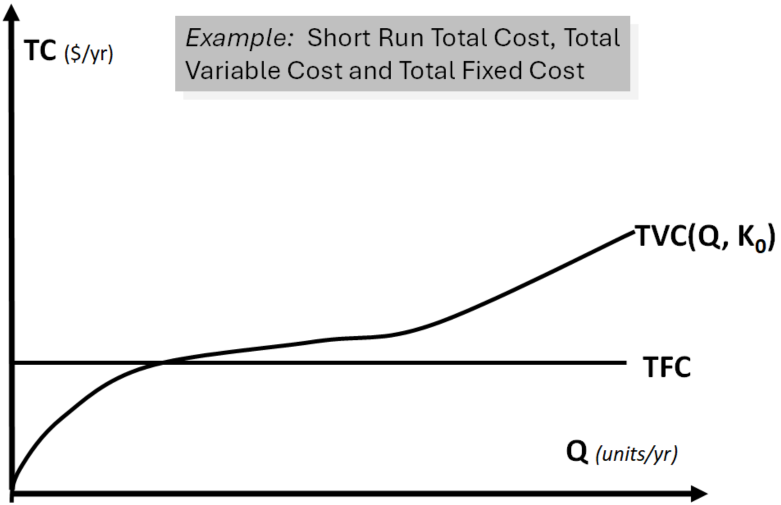
$$STC(Q, K_0) = TVC(Q, K_0) + TFC(Q, K_0)$$

Where: K_0 is the fixed input and w and r are fixed (and suppressed as arguments).

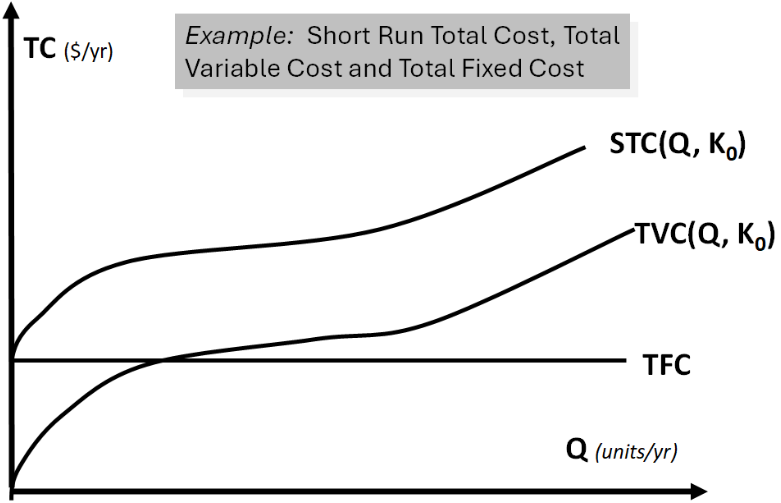
Key Cost Functions Interactions



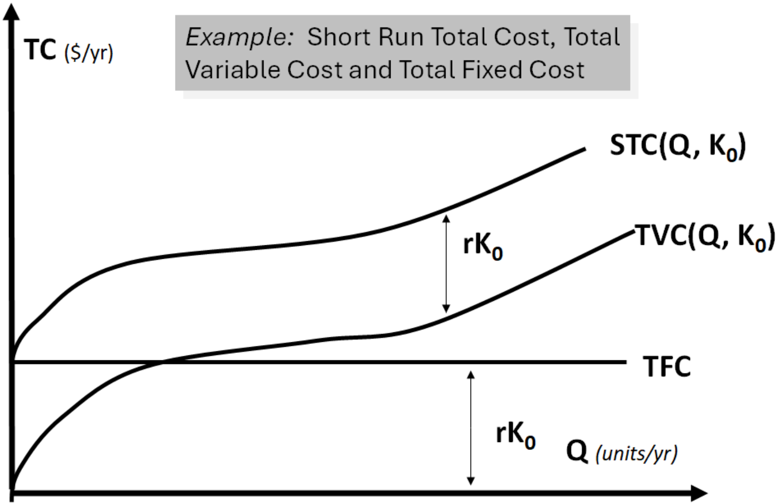
Key Cost Functions Interactions



Key Cost Functions Interactions



Key Cost Functions Interactions



Long and Short Run Total Cost Functions

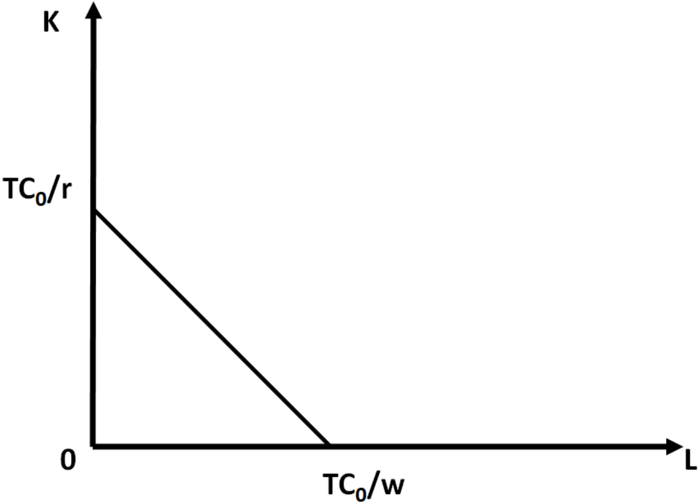
Understanding the Relationship

The firm can minimize costs at least as well in the long run as in the short run because it is *less constrained*.

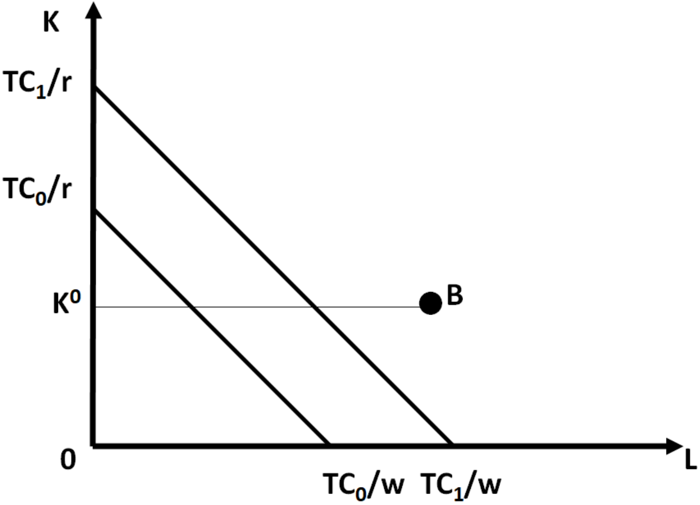
Hence, the short run total cost curve lies everywhere above the long run total cost curve.

However, when the quantity is such that the amount of the fixed inputs just equals the optimal long run quantities of the inputs, the short run total cost curve and the long run total cost curve coincide.

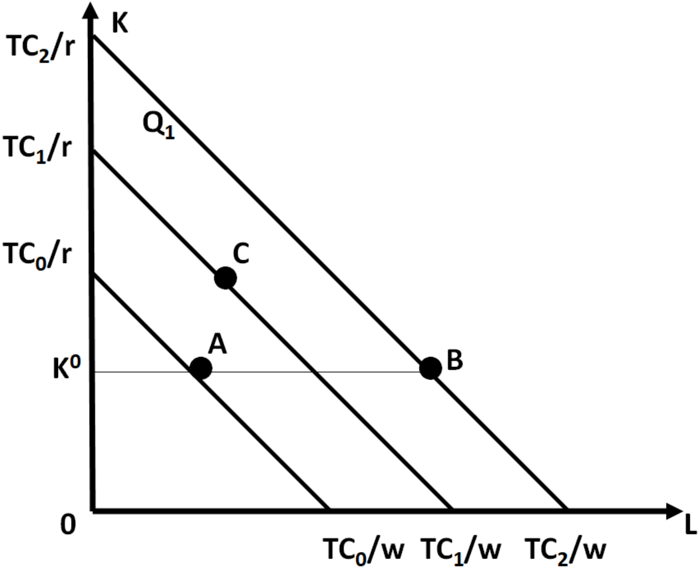
Long and Short Run Total Cost Functions



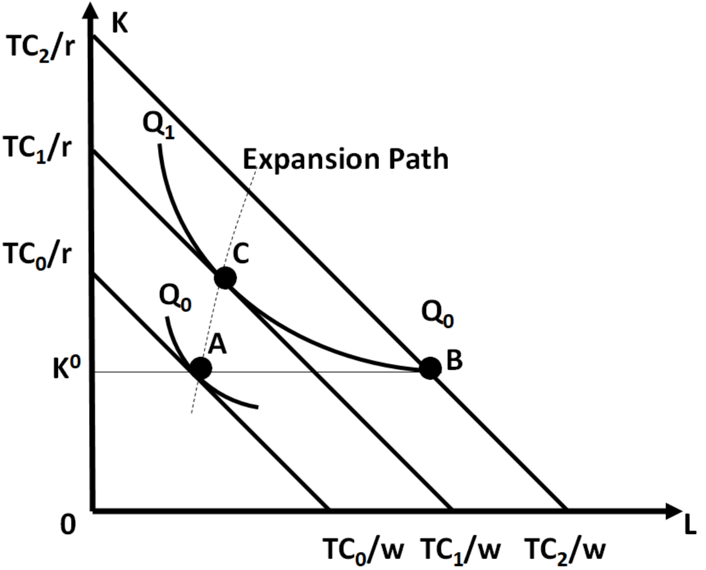
Long and Short Run Total Cost Functions



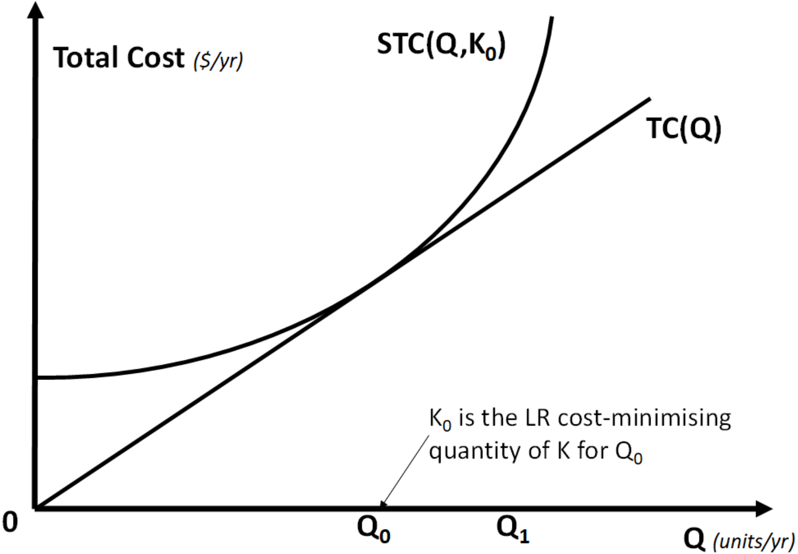
Long and Short Run Total Cost Functions



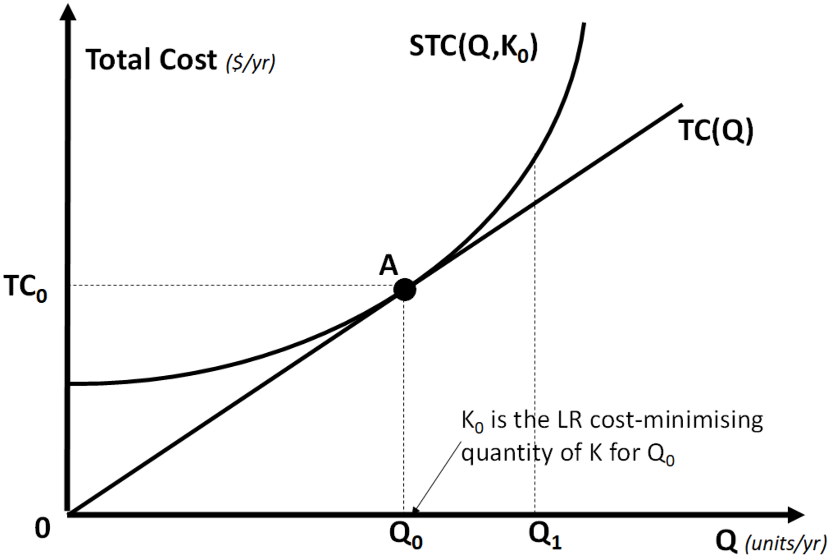
Long and Short Run Total Cost Functions



Long and Short Run Total Cost Functions



Long and Short Run Total Cost Functions



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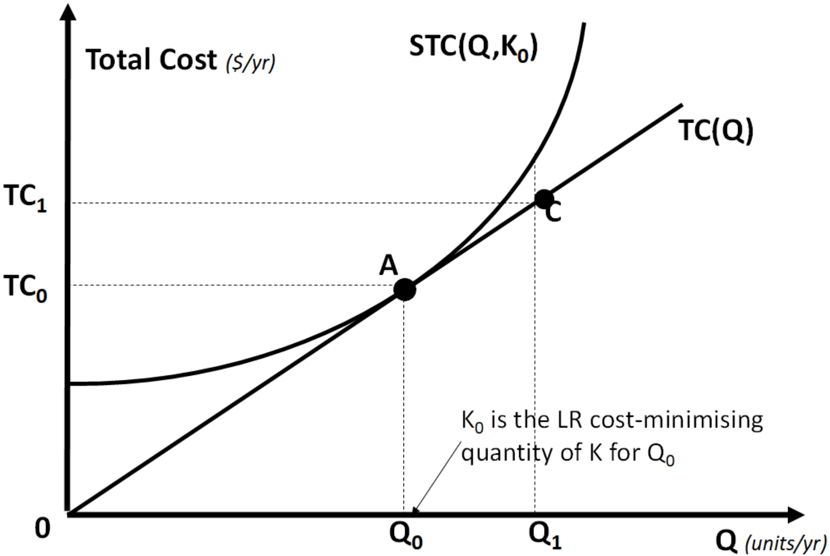
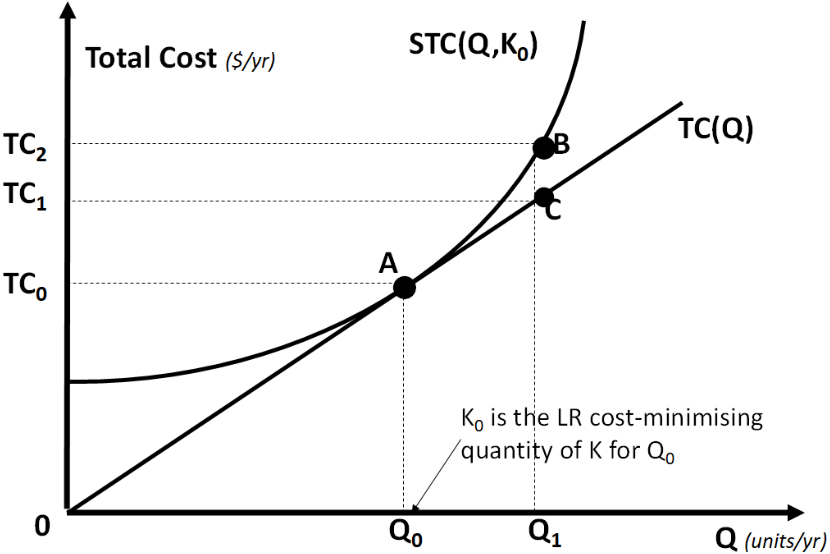


Long and Short Run Total Cost Functions



Short Run Average Cost Functions

Short Run Average Cost Functions

The **short run average cost function** is the short run total cost function divided by output, Q.
That is, the *SAC* function tells us the firm's short run cost per unit of output.

$$SAC(Q, K_0) = \frac{STC(Q, K_0)}{Q}$$

Where: w and r are held fixed.

Short Run Marginal Cost Functions

Short Run Marginal Cost Functions

The **short run marginal cost function** measures the rate of change of short run total cost as output varies, holding constant input prices and fixed inputs.

$$SMC(Q, K_0) = \frac{\Delta STC(Q, K_0)}{\Delta Q}$$

Where: w, r, and K_0 are held constant.

Summary Cost Functions

Note: When $STC = TC$, $SMC = MC$

$$STC = TVC + TCF$$

$$SAC = AVC + ACF$$

Where

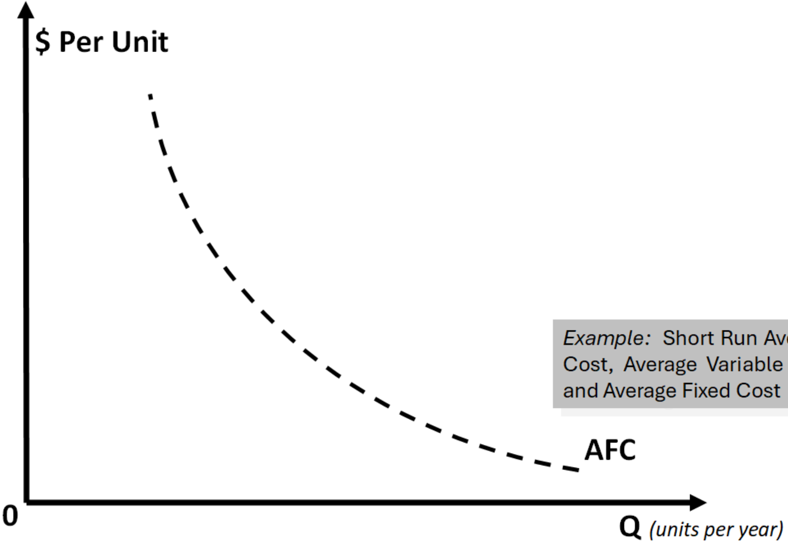
$$SAC = \frac{STC}{Q}$$

$$AVC = \frac{TVC}{Q} \; (average\ variable\ cost)$$

$$AFC = \frac{TFC}{Q} \; (average\ fixed\ cost)$$

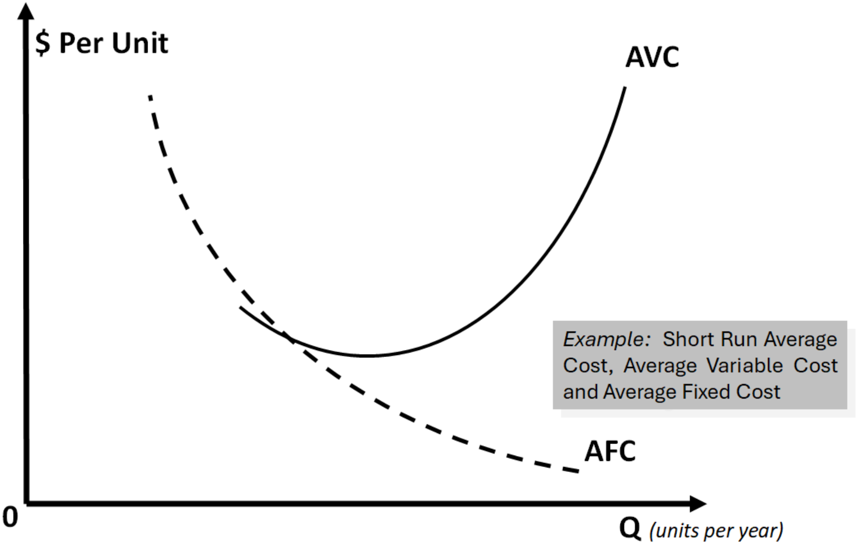
The SAC function is the VERTICAL sum of the AVC and AFC functions.

Summary Cost Functions

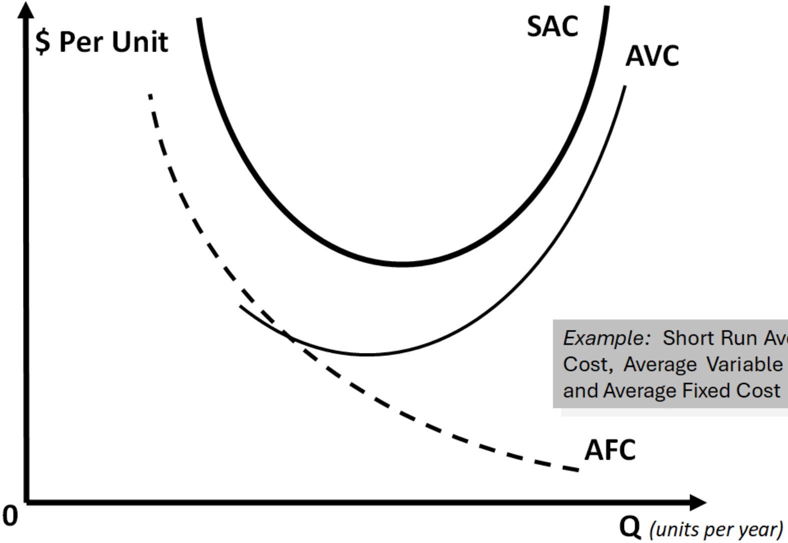


Example: Short Run Average Cost, Average Variable Cost and Average Fixed Cost

Summary Cost Functions

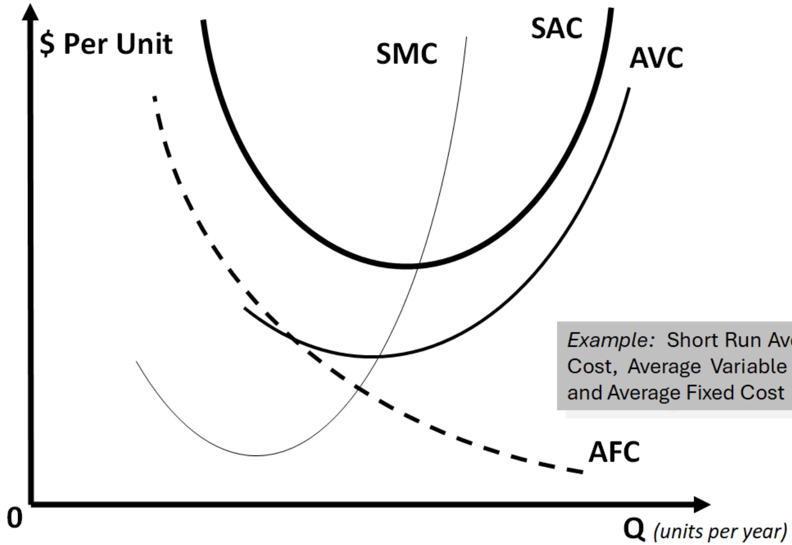


Summary Cost Functions



Example: Short Run Average Cost, Average Variable Cost and Average Fixed Cost

Summary Cost Functions



Example: Short Run Average Cost, Average Variable Cost and Average Fixed Cost

Perfectly Competitive Markets

Adapted from Chapter 9 of Besanko's Microeconomics

Luisa Lorè
Department of Economics
University of Innsbruck

May 30, 2025

Overview

1. Perfect Competition Defined
2. The Profit Maximization Hypothesis
3. The Profit Maximization Condition
4. Short Run Equilibrium
5. Long Run Equilibrium

Perfectly Competitive Markets

Perfectly Competitive Markets

A **perfectly competitive market** consists of firms that produce identical products that sell at the same price.
Each firm's volume of output is so small in comparison to the overall market demand that no single firm has an impact on the market price.

Perfectly Competitive Markets - Conditions

(a) Firms produce **undifferentiated products** in the sense that consumers **perceive** them to be identical

(b) Consumers have **perfect information** about the prices all sellers in the market charge

(c) Each buyer's purchases are so **small** that he/she has an imperceptible effect on market price.

(d) Each seller's sales are so **small** that he/she has an imperceptible effect on market price. Each seller's input purchases are so **small** that he/she perceives no effect on input prices

(e) All firms (*industry participants and new entrants*) have **equal access to resources** (*technology, inputs*).

Implications of Conditions

The Law of one Price

Conditions (a) and (b) imply that there is a single price at which transactions occur.

Price Takers

Conditions (c) and (d) imply that buyers and sellers take the price of the product as given when making their purchase and output decisions.

Free Entry

Condition (e) implies that all firms have identical long-run cost functions

The Profit Maximization Hypothesis

Economic Profit

Economic Profit $=$ Sales Revenue - Economic (Opportunity) Cost

The Profit Maximization Hypothesis

Economic Profit

Economic Profit = Sales Revenue - Economic (Opportunity) Cost

Example

- Revenues: $\$1M$
- Costs of supplies and labor: $\$850,000$
- Owner's best outside offer: $\$200,000$

The Profit Maximization Hypothesis

Economic Profit

Economic Profit = Sales Revenue - Economic (Opportunity) Cost

Example

- Revenues: $\$1M$
- Costs of supplies and labor: $\$850,000$
- Owner's best outside offer: $\$200,000$

Accounting Profit

The Profit Maximization Hypothesis

Economic Profit

Economic Profit = Sales Revenue - Economic (Opportunity) Cost

Example

- Revenues: $\$1M$
- Costs of supplies and labor: $\$850,000$
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Accounting Profit $\$1M - \$850,000 = \$150,000$

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$Accounting\ Profit$ $\$1M - \$850,000 = \$150,000$
$Economic\ Profit$ $\$1M - \$850,000 - \$200,000 = -\$50,000$

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Accounting Profit $\$1M - \$850,000 = \$150,000$
Economic Profit $\$1M - \$850,000 - \$200,000 = -\$50,000$
Business "destroys" \$50,000 of wealth of owner

The Profit Maximization Condition

Assuming the firm sells output Q, its economic profit is:

$$\pi = TR(Q) - TC(Q)$$

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- $TR(Q) =$ Total revenue from selling the quantity $Q \Rightarrow TR(Q) = P \times Q$
- $TC(Q) =$ Total economic cost of producing the quantity Q

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Since P is taken as given, the firm chooses Q to maximize profit. Marginal Revenue: The rate at which TR changes with output.

$$MR = \frac{\Delta TR}{\Delta Q}$$

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$$MR = \frac{\Delta TR}{\Delta Q}$$

Since the firm is a price taker, an increase in TR from a 1-unit change in Q is equal to P.

$$MR = \frac{\Delta TR}{\Delta Q} = \frac{\Delta(P \times Q)}{\Delta Q} = P$$

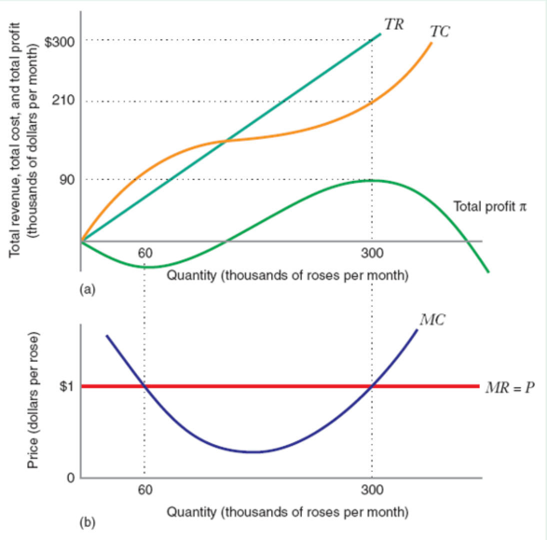
The Profit Maximization Condition

Note

- If $P > MC$, then profit rises if output is increased.
- If $P < MC$, then profit falls if output is increased.

Therefore, the profit maximization condition for a price-taking firm is $P = MC$

The Profit Maximization Condition



The Profit Maximization Condition

At profit maximizing point:

1. $P = MC = MR$
2. MC rising

The Profit Maximization Condition

At profit maximizing point:

1. $P = MC = MR$
2. MC rising

firm demand $= P$ (sells as much as they likes at P)
firm supply defined by MC curve? Not quite.

Short Run Equilibrium

Short Run Equilibrium

For the following, the **short run** is the period of time in which the firm's plant size is fixed and the number of firms in the industry is fixed.

$$STC(Q) = SFC + NSFC + TVC(q) \text{ for } q > 0$$

$$STC(Q) = SFC \text{ for } q = 0$$

Short Run Equilibrium

SFC is the cost of the firm's fixed inputs that are unavoidable at $q = 0$

Output insensitive for $q > 0$ = Sunk

$NSFC$ is the cost of the firm's inputs that are avoidable if the firm produces zero (salaries of some employees, for example)

Output insensitive for $q > 0$ = Non-sunk

$TFC = SFC + NSFC$

$TVC(q)$ are the output sensitive costs *(and are non-sunk)*

Short Run Supply Curve (SRSC)

Short Run Supply Curve (SRSC)

The firm's **short-run supply curve** tells us how the profit-maximizing output changes as the market price changes.

Short Run Supply Curve: $\mathbf{NSFC = 0}$

If the firm chooses to produce a positive output, $P = SMC$ defines the short-run supply curve of the firm. But...

Shutdown Price

The firm will choose to produce a positive output only if:

$$\pi(q) > \pi(0)$$
$$Pq - TVC(q) - TFC > -TFC$$
$$Pq - TVC(q) > 0$$
$$P > AVC(q)$$

Shutdown Price

The price below which the firm would opt to produce zero is called the **shutdown price**, P_s. In this case, P_s is the minimum point on the AVC curve.

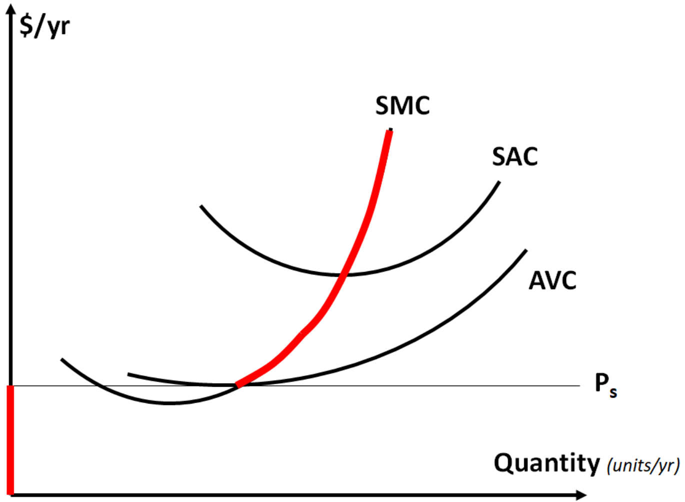
Short Run Supply Function

Therefore, the firm's short-run supply function is defined by:

1. $P = SMC$, where SMC slopes upward as long as $P \geq P_s$
2. 0 where $P < P_s$

This means that a perfectly competitive firm may choose to operate in the short run even if economic profit is negative.

Short Run Supply Curve



Cost Considerations

At prices below SAC but above AVC, profits are negative if the firm produces... but the firm loses less by producing than by shutting down because of sunk costs.

Example

$$STC(q) = 100 + 20q + q^2$$

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$$AVC(q) = 20 + q$$
$$SMC(q) = 20 + 2q$$

Cost Considerations

Example

The minimum level of AVC is the point where $AVC = SMC$ or:

Cost Considerations

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$$20 + q = 20 + 2q$$

$$q = 0$$

AVC minimized at 20

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The firm's short-run supply curve is, then:

Cost Considerations

Example

The minimum level of AVC is the point where $AVC = SMC$ or:

$$20 + q = 20 + 2q$$

$$q = 0$$

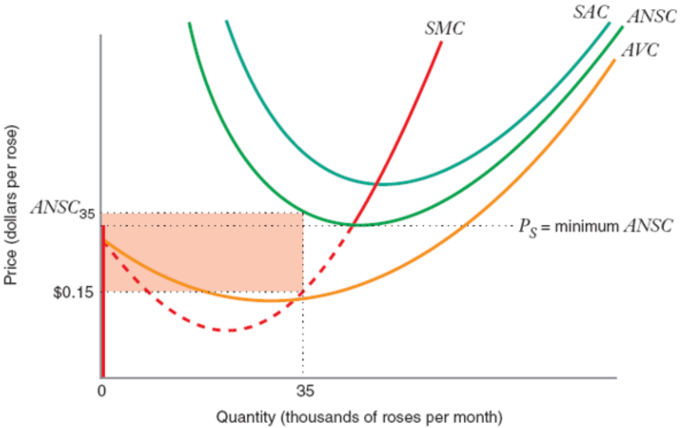
AVC minimized at 20

The firm's short-run supply curve is, then:

$$P < P_s = 20 \Rightarrow q_s = 0$$

$$P \geq P_s = 20 : P = SMC \Leftrightarrow P = 20 + 2q \Leftrightarrow q_s = 10 + \frac{1}{2}P$$

SRSC When Some Costs are Sunk and Some are Non-Sunk



$$TFC = SFC + NSFC, \text{ where } NSFC > 0$$
$$ANSC = AVC + \frac{NSFC}{Q}$$

Now, the shut-down price, P_s is the minimum of the $ANSC$ curve.

SRSC When All Costs are Non-Sunk

If the firm chooses to produce a positive output, $P = SMC$ defines the short-run supply curve of the firm. But the firm will choose to produce a positive output only if:

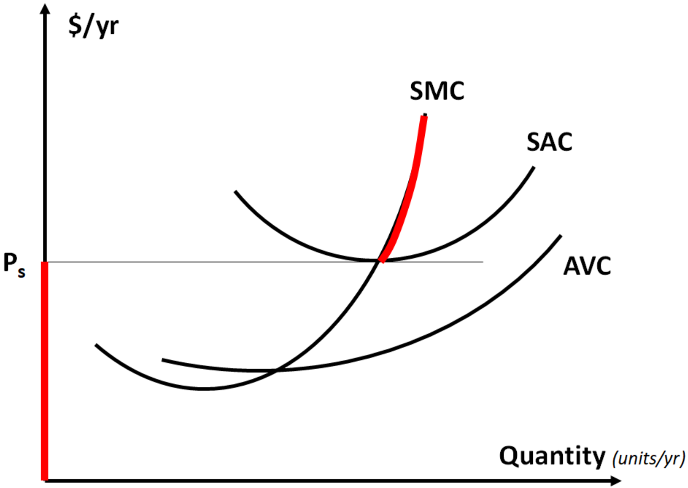
$$\pi(q) \geq pi(0)$$

$$Pq - TVC(q) - TFC > 0$$

$$P > AVC(q) + AFC(q) = SAC(q)$$

Now, the shutdown price, P_s, is the minimum of the SAC curve.

SRSC When All Costs are Non-Sunk



SRSC When All Costs are Non-Sunk

Example

$$STC(q) = F + 20q + q^2$$

$F = 100$, all of which is sunk:

$$AVC(q) = 20 + q$$

$$SMC(q) = 20 + 2q$$

$$SAC(q) = \frac{100}{q} + 20 + q$$

$SAC = SMC$ at $q = 10$

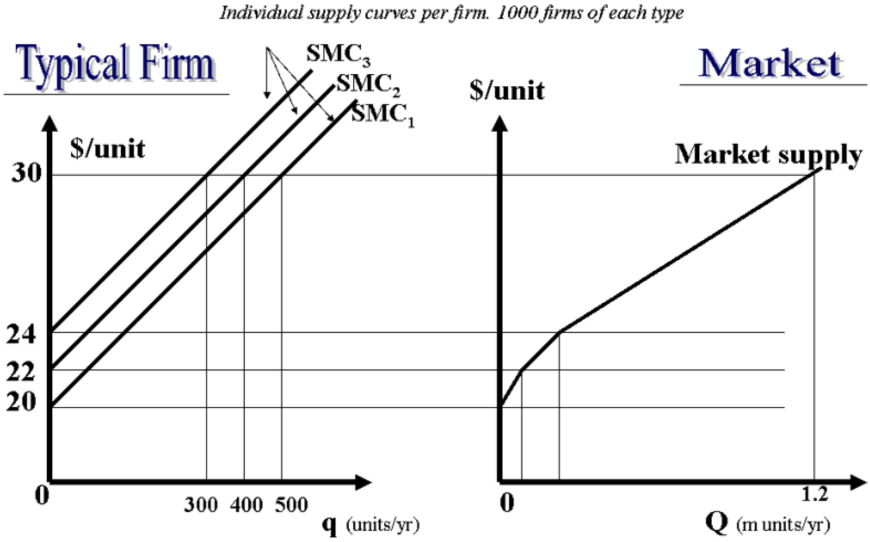
At any $P > 40$, the firm earns positive economic profit.
At any $P < 40$, the firm earns negative economic profit.

Market Supply

Market Supply

The **market supply** at any price is the sum of the quantities each firm supplies at that price.
The short-run market supply curve is the horizontal sum of the individual firm supply curves.

Short Run Market & Supply Curves



Short Run Perfectly Competive Equilibrium

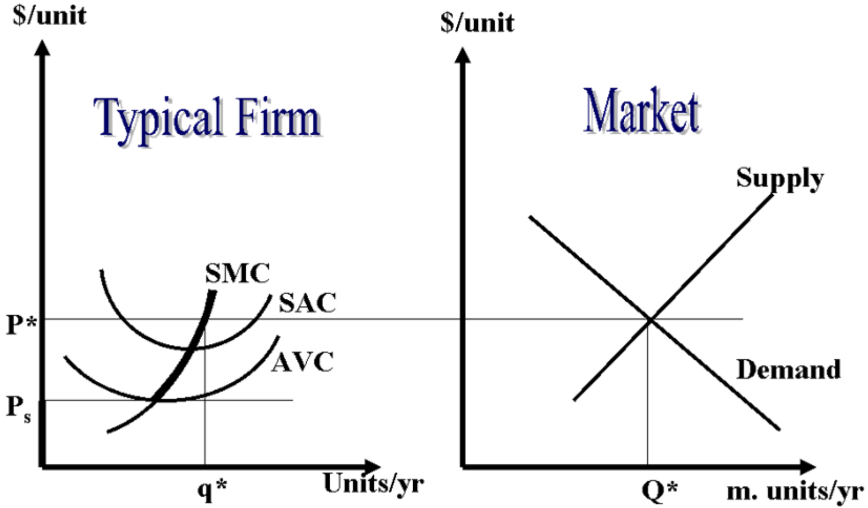
Short Run Perfectly Competive Equilibrium

A short-run perfectly competitive equilibrium occurs when the market quantity demanded equals the market quantity supplied.

$$\Sigma_{i=1}^{n} = Q_s^i(P) = Q_d(P)$$

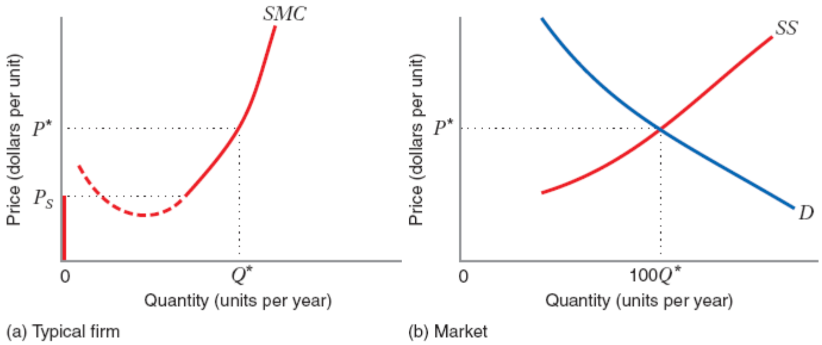
and $Q_s^i(P)$ is determined by the firm's individual profit maximization condition.

Short Run Perfectly Competive Equilibrium



Short Run Market Equilibrium

- Short-run perfectly competitive equilibrium: The market price at which quantity demanded equals quantity supplied.
- A typical firm produces Q^* where $MR = MC$, and if 100 firms make up the market then market supply must equal $100Q^*$



Deriving a Short Run Market Equilibrium

Example

300 Identical Firms

$$Q_d(P) = 60 - P$$

$$STC(q) = 0.1 + 150q^2$$

$$SMC(q) = 300q$$

$$NSFC = 0$$

$$AVC(q) = 150q$$

Minimum $AVC = 0$ so as long as the price is positive, the firm will produce.

Deriving a Short Run Market Equilibrium

Example

Short Run Equilibrium

$$\text{Profit maximization condition: } P = 300q$$

$$q_s(P) = \frac{P}{300} \text{ and } Q_s(P) = 300\left(\frac{P}{300}\right) = P$$

$$Q_s(P) = Q_d(P) \Rightarrow P = 60 - P$$

$$P^* = 30$$

$$q^* = \frac{30}{300} = 0.1$$

$$Q^* = 30$$

Deriving a Short Run Market Equilibrium

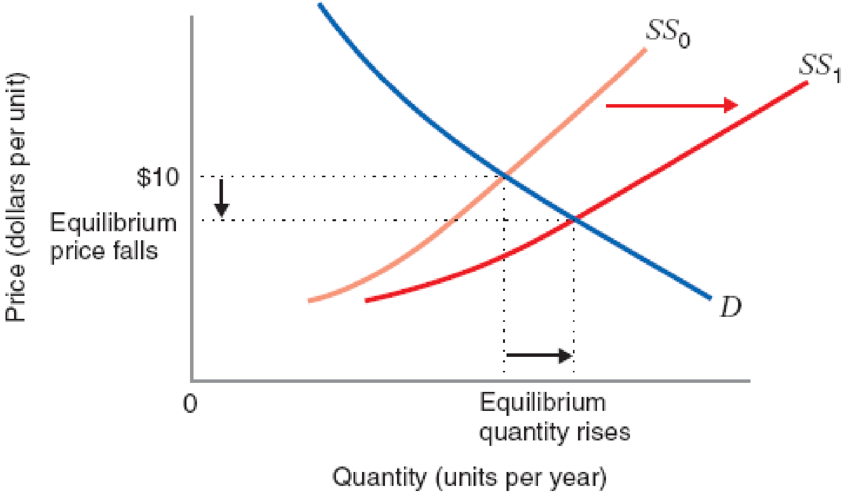
Example

Do firms make positive profits at the market equilibrium?

$$SAC = \frac{STC}{q} = \frac{0.1}{q} + 150q$$

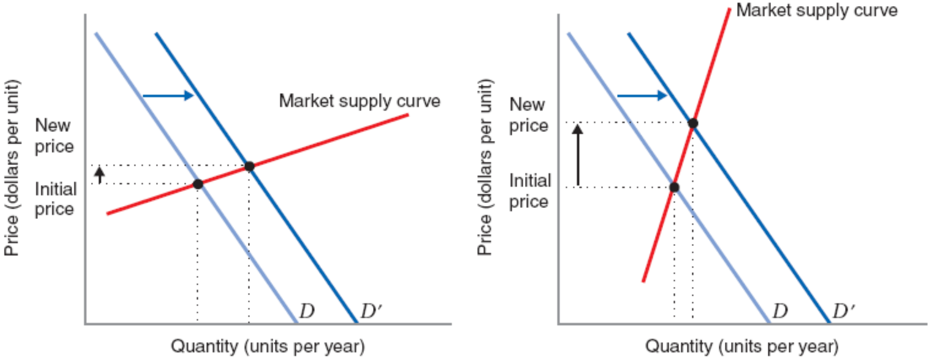
When each firm produces .1, SAC per firm is: $\frac{0.1}{0.1} + 150(0.1) = 16$
Therefore, $P^* > SAC$ so profits are positive

Comparative Statics



If Supply shifts when the number of firms increases.

Comparative Statics



(a) Effect of shift in demand: Supply is relatively elastic

(b) Effect of shift in demand: Supply is relatively inelastic

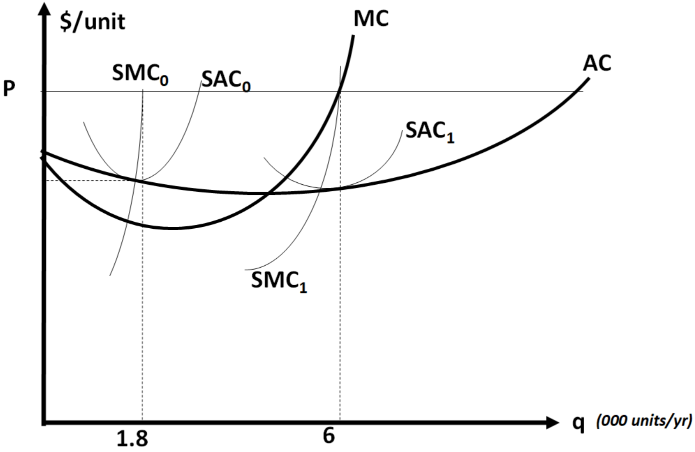
When demand shifts, the elasticity of supply matters.

Long Run Market Equilibrium

Long Run Market Equilibrium

For the following, the **long run** is the period of time in which all the firm's inputs can be adjusted. The number of firms in the industry can change as well.
The firm should use long run cost functions for evaluating the cost of outputs it might produce in this longer term period. . . i.e., decisions to modify plant size, enter or exit, change production process and so on would all be based on long term analysis

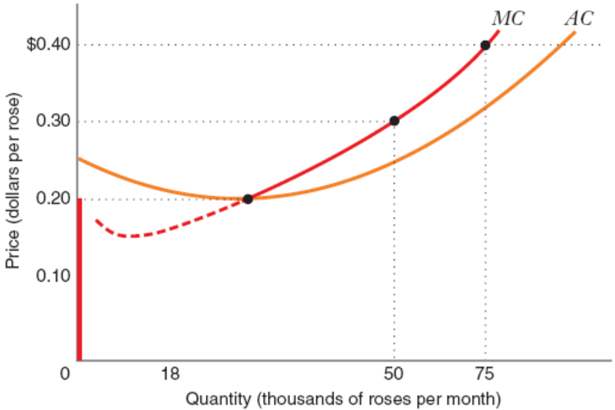
Long Run Market Equilibrium



Example: Incentive to Change Plant Size

For example, at P, this firm has an incentive to change plant size to level K_1 from K_0.

Firm's Long Run Supply Curve



The firm's long run supply curve:

$$P = MC \text{ for } P > (min(AC) = P_s)$$

$$0(exit) \text{ for } P < (min(AC) = P_s)$$

For prices greater that \$0.20 the long-run supply curve is the long-run MC curve.

Long Run Market Equilibrium

A long run perfectly competitive equilibrium occurs at a market price, P^*, a number of firms, n^*, and an output per firm, q^* that satisfies:

Long Run Profit Maximization

Long run profit maximization with respect to output and plant size:

$$P^* = MC(q^*)$$

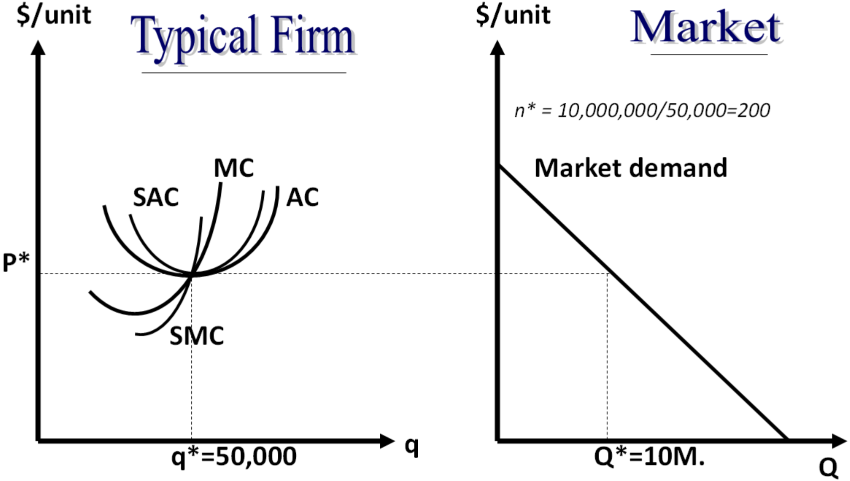
Zero economic profit

$$P^* = AC(q^*)$$

Demand equals supply

$$Q^d(P^*) = n^* q^* \text{ or } n^* = \frac{Q^d(P^*)}{q^*}$$

Long Run Perfectly Competitive



Calculating Long Run Equilibrium

Example

$$Q^d(P) = 25000 - 1000P$$

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$$Q^d(P) = 25000 - 1000P$$
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Calculating Long Run Equilibrium

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The long run equilibrium satisfies the following:

Calculating Long Run Equilibrium

Example

$$Q^d(P) = 25000 - 1000P$$
$$TC(q) = 40q - q^2 + .01q^3$$
$$AC(q) = 40 - q + .01q^2$$
$$MC(q) = 40 - 2q + .03q2$$

The long run equilibrium satisfies the following:

(a) $P^* = MC(q) \Rightarrow P^* = 40 - 2q^* - .03q^*2$

(b) $P^* = AC(q) \Rightarrow P^* = 40 - q + .01q^2$

(c) $Q^d(P^*) = n^*q^* \Rightarrow 25000 - 1000P^* = n^*q^*$

Calculating Long Run Equilibrium

Example

Using (a) and (b), we have:

Calculating Long Run Equilibrium

Example

Using (a) and (b), we have:

$$40–2q^* + .03q^{*2} = 40 - q^* + .01q^{*2}$$

Calculating Long Run Equilibrium

Example

Using (a) and (b), we have:

$$40–2q^* + .03q^{*2} = 40 - q^* + .01q^{*2}$$

$$q^* = 50$$

$$P^* = 15$$

$$Q^d(P^*) = 10000$$

Calculating Long Run Equilibrium

Example

Using (a) and (b), we have:

$$40–2q^* + .03q^{*2} = 40 - q^* + .01q^{*2}$$

$$q^* = 50$$

$$P^* = 15$$

$$Q^d(P^*) = 10000$$

Using (c) we have:

Calculating Long Run Equilibrium

Example

Using (a) and (b), we have:

$$40–2q^* + .03q^{*2} = 40 - q^* + .01q^{*2}$$

$$q^* = 50$$

$$P^* = 15$$

$$Q^d(P^*) = 10000$$

Using (c) we have:

$$n^* = \frac{10000}{50} = 200$$

Calculating Long Run Equilibrium

Summarizing long run equilibrium: *If anyone can do it, you can't make money at it*

Or if the firm's strategy is based on skills that can be easily imitated or resources that can be easily acquired, in the long run, your economic profit will be competed away.

Long Run Market Supply Curve

We have calculated a point at which the market will be in long-run equilibrium. This is a point on the long-run market supply curve. This curve can be derived explicitly, however.

Long Run Market Supply Curve

The **Long Run Market Supply Curve** tells us the total quantity of output that will be supplied at various market prices, assuming that all long run adjustments (plant, entry) take place.

Long Run Market Supply Curve

Since new entry can occur in the long run, we cannot obtain the long run market supply curve by summing the long run supplies of current market participants.

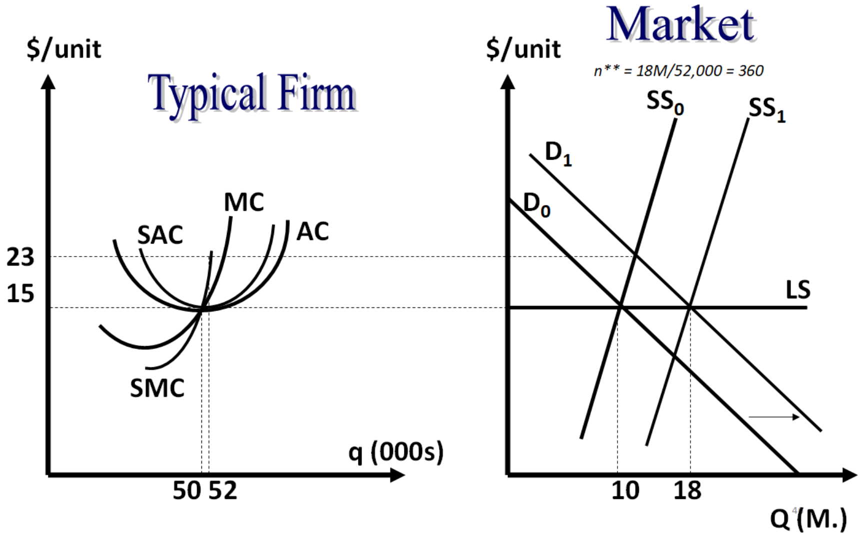
Instead, we must construct the long-run market supply curve.

We reason that, in the long run, output expansion or contraction in the industry occurs along a horizontal line corresponding to the minimum level of long-run average cost.

If $P > min(AC)$, entry would occur, driving price back to $min(AC)$.

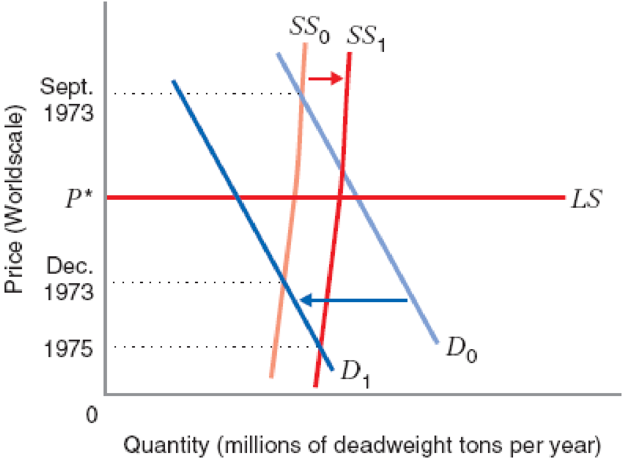
If $P < min(AC)$, firms would earn negative profits and would supply nothing.

Long Run Market Supply Curve



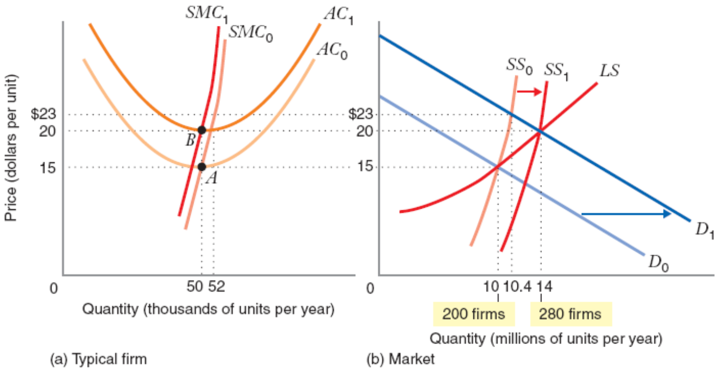
Constant Cost Industry

Constant-cost Industry: An industry in which the increase or decrease of industry output does not affect the price of inputs.

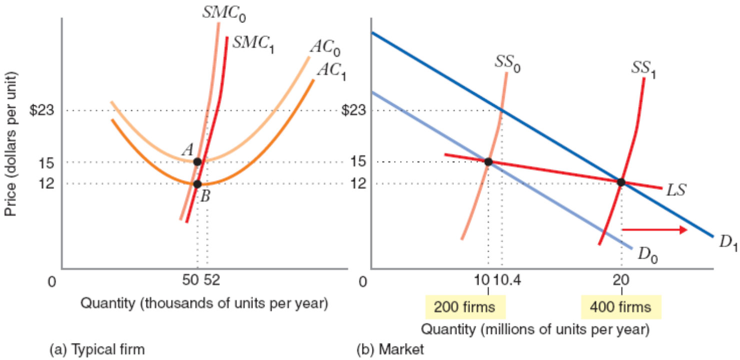


Increasing Cost Industry

Increasing cost Industry: An industry which increases in industry output increase the price of inputs. Especially if firms use industry specific inputs i.e. scarce inputs that are used only by firms in a particular industry and no other industry.



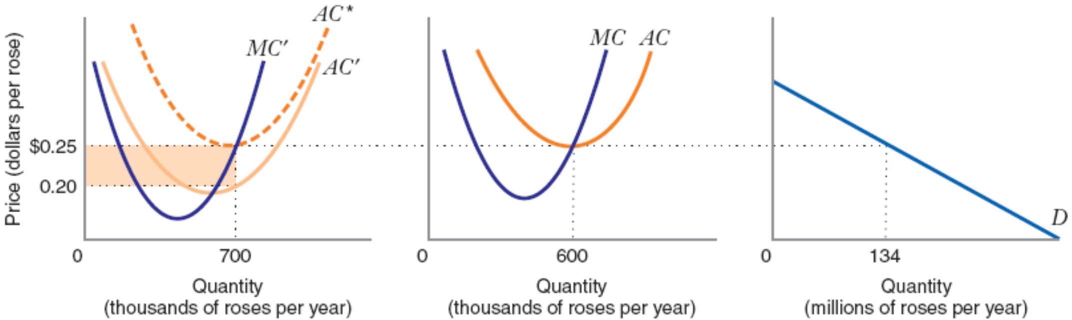
Decreasing-cost Industry: An industry in which increases in industry output decrease the prices of some or all inputs.



Economic Rent

- **Economic Rent:** The economics rent that is attributed to extraordinarily productive inputs whose supply is scarce.
  $\rightarrow$ Difference between the maximum value is willing to pay for the services of the input and input's reservation value.
- **Reservation value:** The returns that the owner of an input could get by deploying the input in its best alternative use outside the industry.

Economic Rent



(a) Rose firm with extraordinary master grower

(b) Rose firm with run-of-the-mill master grower

(c) Market for roses

Producer Surplus

Producer Surplus

Producer Surplus is the area above the market supply curve and below the market price. It is a monetary measure of the benefit that producers derive from producing a good at a particular price.

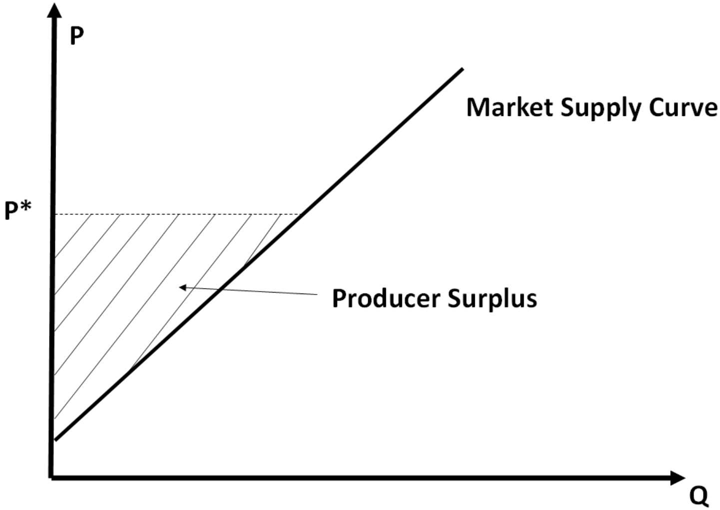
Note that the producer earns the price for every unit sold, but only incurs the SMC for each unit. This is why the difference between the P and SMC curve measures the total benefit derived from production.

Producer Surplus

Further, since the market supply curve is simply the sum of the individual supply curves... which equal the marginal cost curves the difference between price and the market supply curve measures the surplus of all producers in the market.

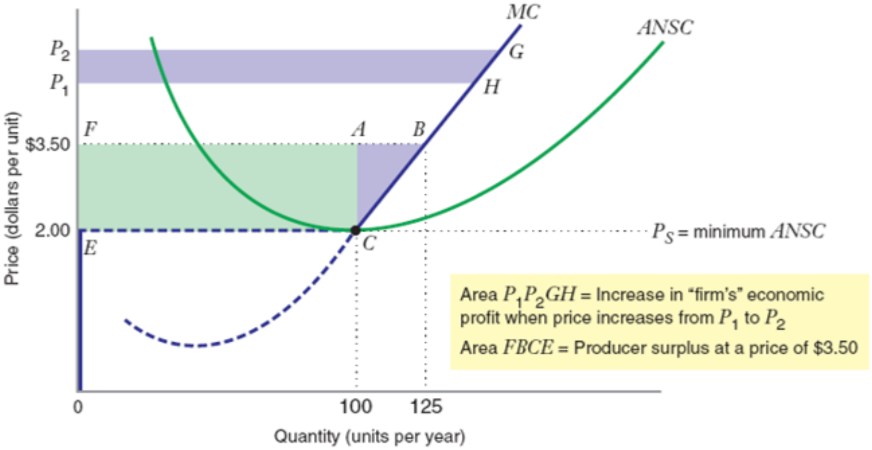
Note that producer's surplus does not deduct fixed costs, so it does not equal profit.

Producer Surplus

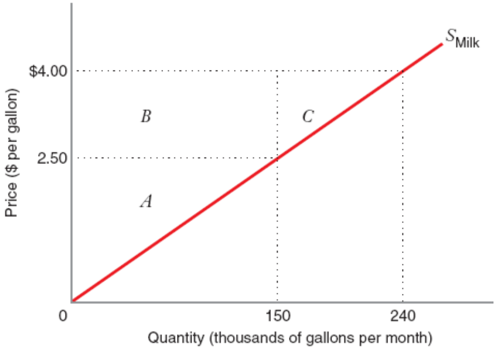


Producer Surplus

- Producer surplus is area *FBCE* when price is \$3.50
- Change in producer surplus is area $P_1 P_2 GH$ when price moves from P_1 to P_2.



Producer Surplus



- Given Market supply curve and P is the price in dollars per gallon
- Find producer surplus when price is $2.50 per gallon
- How much does producer surplus when price of milk increases from $2.50 to $4.00

Producer Surplus

$$Q = 60(2.50) = 150$$
$$\text{Area } A = \frac{1}{2}(2.5 - 0)(150000) = 187,500$$
$$\text{Area } B = 225,000$$
$$\text{Area } C = 67,500$$
$$\text{Producer Surplus} = 292,000 \text{ per month}$$

- When the price is \$2.50 per gallon, 1,50,000 gallons of milk are sold per month.
- Producer surplus is triangle A
- Price increases from \$2.50 to \$4.00 the quantity supplied will increase to 240,000 gallons per month
- Producer surplus will increase by areas B and area C

Competitive Markets: Applications

Adapted from Chapter 10 of Besanko's Microeconomics

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June 11, 2025

Overview

1. Motivation
2. Deadweight Loss
3. Government Intervention
4. Example of various government policies

Economic Effiency

Economic Effiency

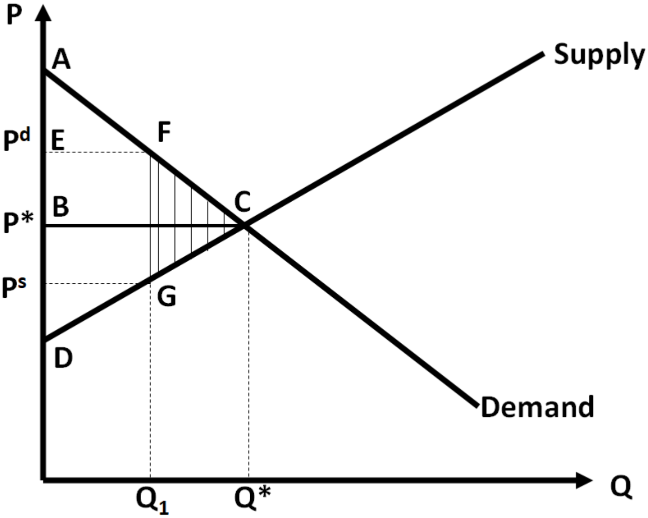
Economic Efficiency means that the total surplus is maximized.

Every consumer who ***is*** *willing to pay more than the opportunity cost of the resources needed to produce extra output is able to buy; every consumer who is* ***not*** *willing to pay the opportunity cost of the extra output does not buy.*

All gains from trade (between buyers and suppliers) are exhausted at the efficient point.

The perfectly competitive equilibrium attains economic efficiency.

Surplus Maximization in Competitive Equilibrium



Surplus Maximization in Competitive Equilibrium

At the Perfectly Competitive Equilibrium, (Q^*, P^*), Total Surplus is maximized.

Consumer's Surplus at (Q^*, P^*) : ABC
Producer's Surplus at (Q^*, P^*) : DBC
Total Surplus at (Q^*, P^*) : ADC

Deadweight Loss

Deadweight Loss

A **deadweight loss** is a reduction in net economic benefits resulting from an inefficient allocation of resources.

Consumer's Surplus at (Q_1, P^d) : AEF
Producer's Surplus at (Q_1, P^D) : $EFGD$
Total Surplus at (Q_1, P^D) : $AFGD$
Deadweight Loss at (Q_1, P^d) : AFC

Government Intervention: Winners & Losers

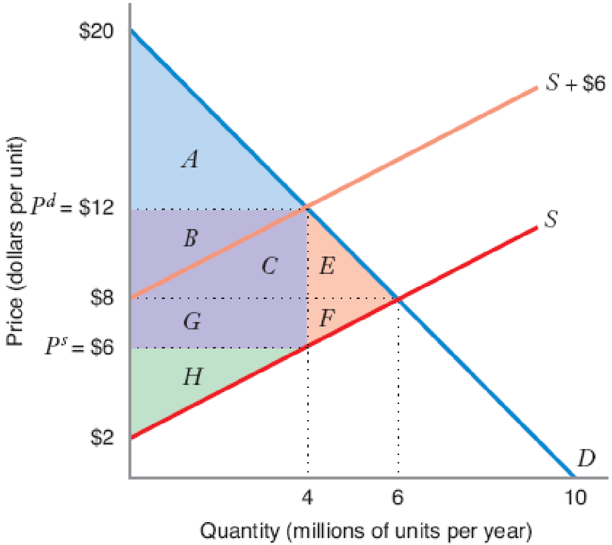
Intervention Type	Effect on (domestic) quantity traded	Effect on (domestic) Consumer Surplus	Effect on (domestic) Producer Surplus	Effect on (domestic) Government Budget	Is a (domestic) Deadweight Loss created?
Excise Tax	Falls	Falls	Falls	Positive	Yes
Subsidies to Producers	Rises	Rises	Rises	Negative	Yes
Maximum Price Ceilings for Producers	Falls; Excess Demand	Rise or Fall	Falls	Zero	Yes
Minimum Price Floors for Producers	Falls; Excess Supply	Falls	Rise or Fall	Zero	Yes
Production Quotas	Falls; Excess Supply	Falls	Rise or Fall	Zero	Yes
Import Tariffs	Falls	Falls	Rises	Positive	Yes
Import Quotas	Falls	Falls	Rises	Zero	Yes

Policy: Excise Tax

Excise Tax

An **excise tax** (or a ***specific tax***) is an amount paid by either the consumer or the producer per unit of the good at the point of sale.
(The amount paid by the demanders exceeds the total amount received by the sellers by amount T)

Policy: Excise Tax



Area	Size (dollars/year)
A	$16 million
B	8 million
C	8 million
E	4 million
F	2 million
G	8 million
H	8 million

Policy: Excise Tax

	With No Tax	With Tax	Impact of Tax
Consumer Surplus	A + B + C + E	A	- B - C - E
Producer Surplus	F + G + H	H	- F - G
Government Receipts from Tax	Zero	B + C + G	B + C + G
Net Benefits	A + B + C + E + F + G + H	A + B + C + G + H	- E – F
Deadweight Loss	Zero	E + F	E + F

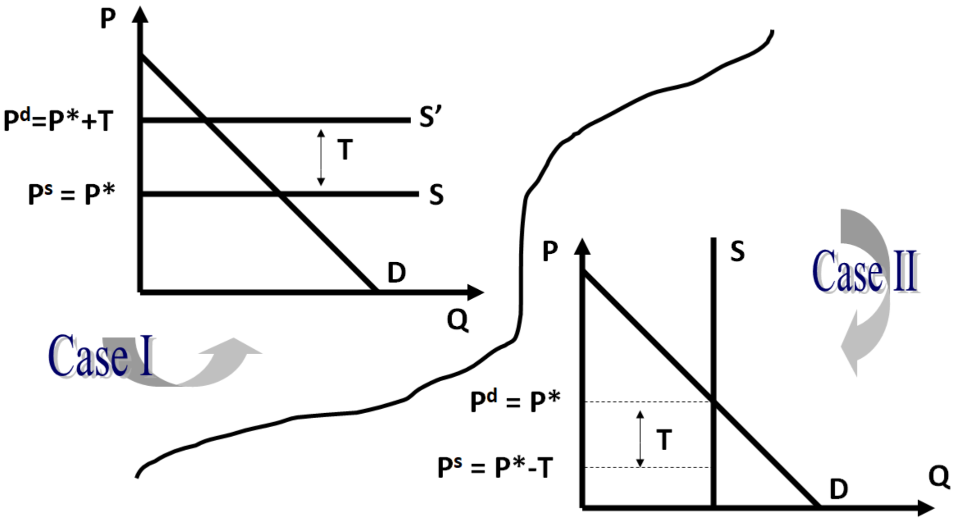
Key Definitions

Incidence of a tax

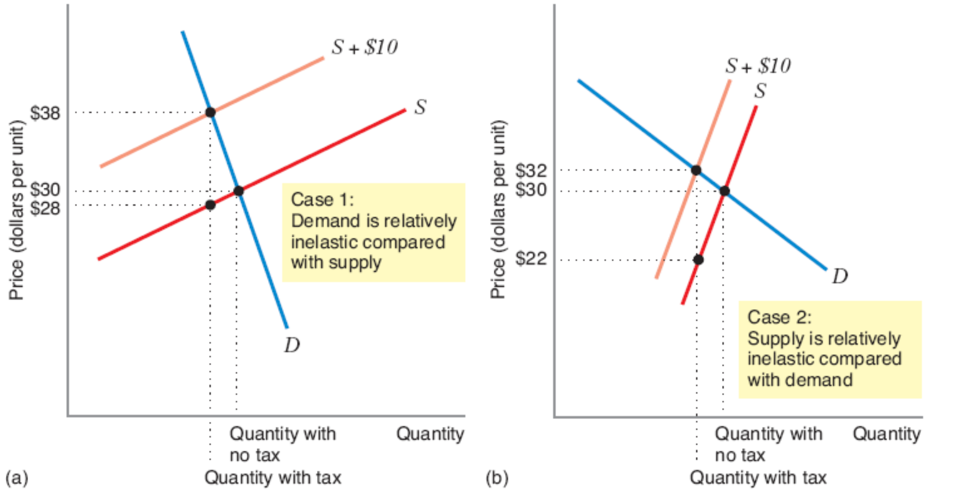
Incidence of a tax is a measure of the effect of a tax on the prices consumers pay and sellers receive in a market.

The amount by which the price paid by buyers, P^d, rises over the non-tax equilibrium price, P^*, is the **incidence of the tax on consumers**; the amount by which the price received by sellers, P^s, falls below P^* is called the **incidence of the tax on producers**.

Incidence of Tax in Two Extreme Cases



Incidence of Tax in Two Cases



Back on the Envelope

Back of the Envelope method to calculate the incidence of a specific tax.

$$\frac{\Delta P^d}{\Delta P^s} = \frac{\eta}{\varepsilon}$$

Where: η is the own-price elasticity of supply ε is the own-price elasticity of demand.

Back of the Envelope

Consider a small tax applied to an economy at point (Q^*, P^*)

$$\varepsilon = \frac{\frac{\Delta Q}{Q^*}}{\frac{\Delta P^d}{P^*}} \Leftrightarrow \frac{\Delta Q}{Q^*} = \varepsilon \frac{\Delta P^d}{P^*}$$

$$\eta = \frac{\frac{\Delta Q}{Q^*}}{\frac{\Delta P^s}{P^*}} \Leftrightarrow \frac{\Delta Q}{Q^*} = \eta \frac{\Delta P^s}{P^*}$$

but for market to clear, $\frac{\Delta Q}{Q^*}$ must be the same for demand and supply, hence

$$\varepsilon \frac{\Delta P^d}{P^*} = \eta \frac{\Delta P^s}{P^*} \Leftrightarrow \varepsilon \Delta P^d = \eta \Delta P^s$$

$$\frac{\Delta P^d}{\Delta P^s} = \frac{\eta}{\varepsilon}$$

Tax Effect

Example

Let $\varepsilon = -0.5$ and $\eta = 2$. What is the relative incidence of a specific tax on consumers and producers?

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Tax Effect

Example

Let $\varepsilon = -0.5$ and $\eta = 2$. What is the relative incidence of a specific tax on consumers and producers?

$$\frac{\Delta P^d}{\Delta P^s} = \frac{\eta}{\varepsilon}$$

$$\frac{\Delta P^d}{\Delta P^s} = \frac{2}{-0.5} = 4$$

Tax Effect

Example

Let $\varepsilon = -0.5$ and $\eta = 2$. What is the relative incidence of a specific tax on consumers and producers?

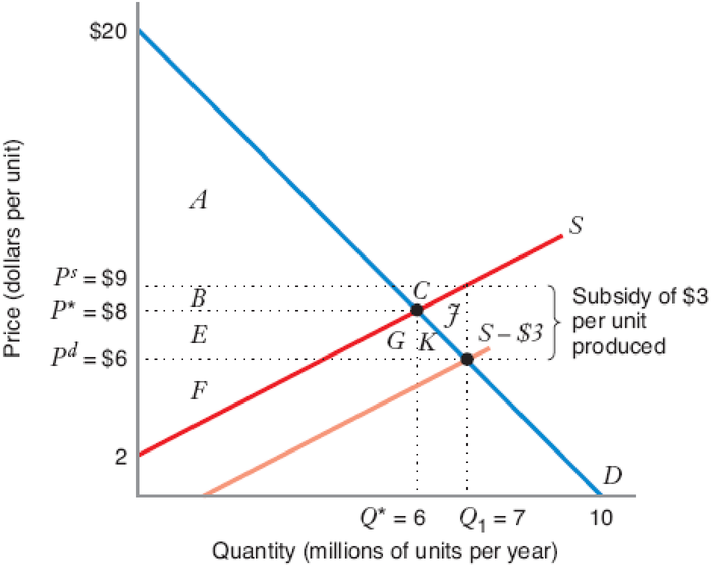
$$\frac{\Delta P^d}{\Delta P^s} = \frac{\eta}{\varepsilon}$$

$$\frac{\Delta P^d}{\Delta P^s} = \frac{2}{-0.5} = 4$$

Interpretation: consumers pay four times as much as the decrease in price producers receive. Hence, an excise tax of \$1 results in an increase in consumer price of \$0.8 and a decrease in price received by producers of \$0.2.

Note: Subsidies are negative taxes.

Subsidies



$20
A
S
$P^s = \$9$
$P^* = \$8$
$P^d = \$6$
B
C
J
E
G
K
S – $3
Subsidy of $3
per unit
produced
F
2
D
$Q^* = 6$
$Q_1 = 7$
10
Price (dollars per unit)
Quantity (millions of units per year)

Subsidies

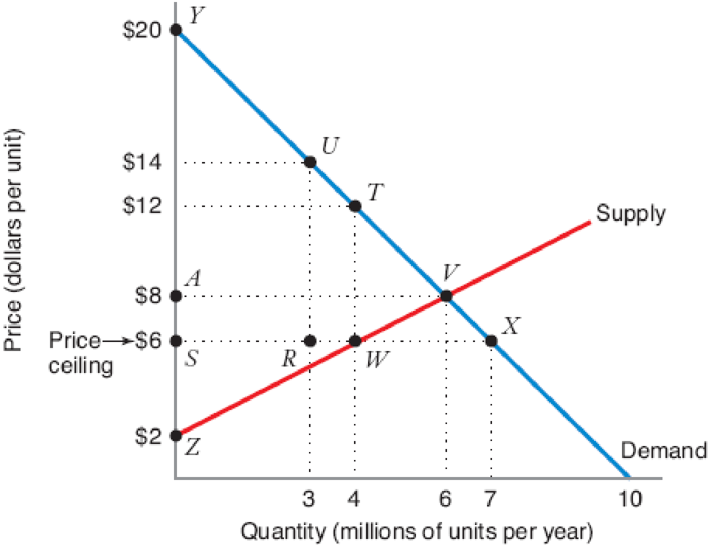
	With No Subsidy	With Subsidy	Impact of Subsidy
Consumer Surplus	A + B	A + B + E + G + K	- B - C - E
Producer Surplus	E + F	B + C + E + F	- F - G
Impact on Government Budget	Zero	- B - C - E - G - K - J	B + C + G
Net Benefits	A + B + E + F	A + B + E + F – J	- E - F
Deadweight Loss	Zero	J	J

Policy: Price Ceilings

Price Ceilings

A **price ceiling** is a legal maximum on the price per unit that a producer can receive. If the price ceiling is *below* the pre-control competitive equilibrium price, then the ceiling is called **binding**.

Policy: Price Ceilings



Policy: Price Ceilings

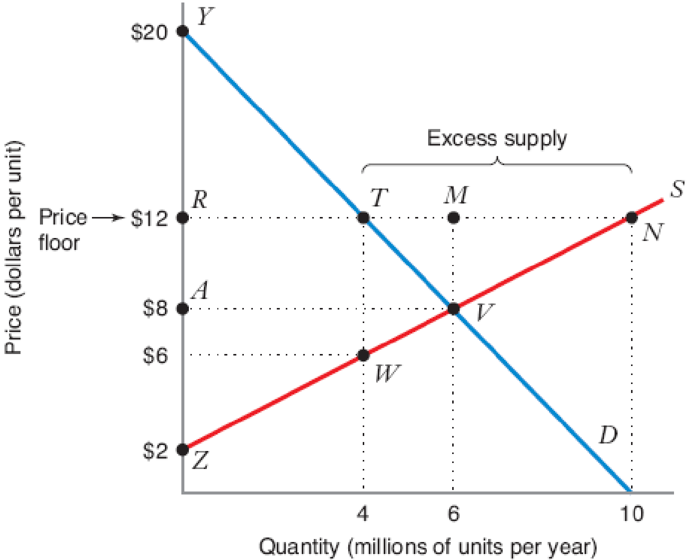
	With No Price Ceiling	With Price Ceiling	
		With Maximum Consumer Surplus	With Minimum Consumer Surplus
Consumer Surplus	Area YAV	Area YTWS	Area URX
Producer Surplus	Area AVZ	Area SWZ	Area SWZ
Net Benefits	Area YZV	Area YTWZ	Areas URX + SWZ
Deadweight Loss	Zero	Area TWV	Area YZV – Area URX – Area SWZ

Policy: Price Floor

Price Floor

A **price floor** is a minimum price that consumers can legally pay for a good. Price floors sometimes are referred to as **price supports**. If the price floor is *above* the pre-control competitive equilibrium price, it is said to be **binding**.

Policy: Price Floor



Policy: Price Floor

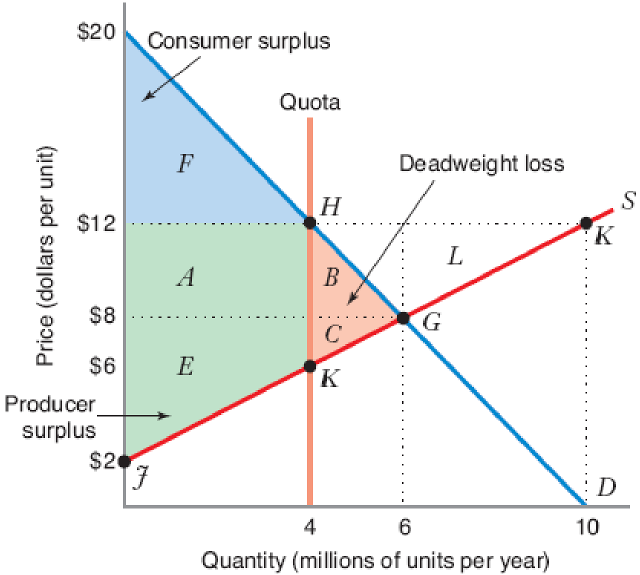
	With No Price Floor	With Price Floor	
		With Maximum Producer Surplus	**With Minimum Producer Surplus**
Consumer Surplus	Area YAV	Area YTR	Area YTR
Producer Surplus	Area AVZ	Area RTWZ	Area MNV
Net Benefits	Area YZV	Area YTWZ	Areas YTR + MNV
Deadweight Loss	Zero	Area TWV	Area YZV – Area YTR – Area MNV

Policy: Production Quotas

Production Quotas

A **production quota** is a limit on either the number of producers in the market or on the amount that each producer can sell. The quota usually has a goal of placing a limit on the total quantity that producers can supply to the market.

Policy: Production Quotas



Policy: Production Quotas

	With No Quota	With Quota	Impact of Quota
Consumer Surplus	A + B + F	F	- A - B
Producer Surplus	C + E	A + E	A - C
Net Benefits	A + B + C + E + F	A + E + F	- B - C
Deadweight Loss	Zero	B + C	B + C

Policy: Import Tariffs & Quotas

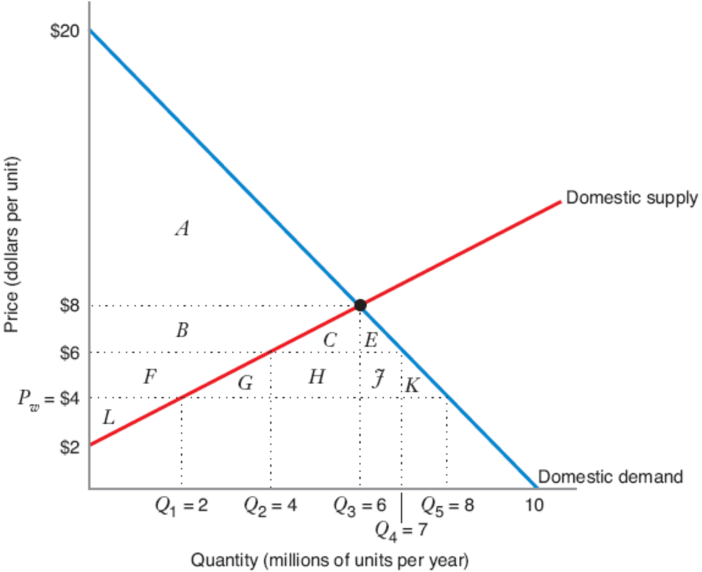
Tariffs

Tariffs are taxes levied by a government on goods imported into the government's own country. Tariffs sometimes are called **duties**.

Import quota

An **import quota** is a limit on the total number of units of a good that can be imported into the country.

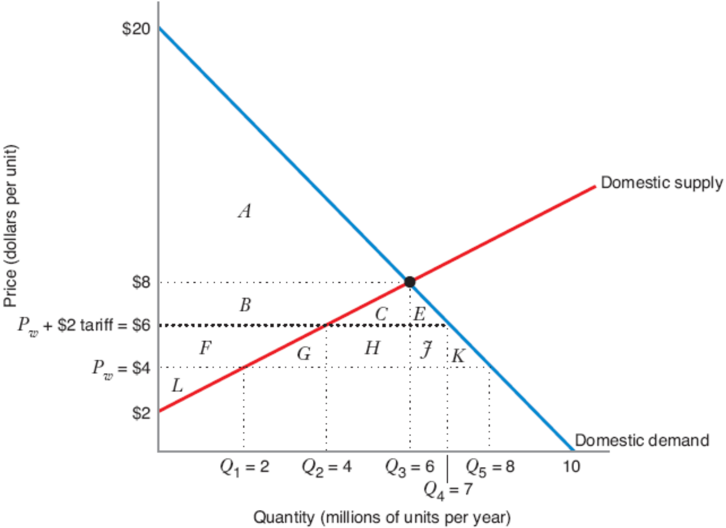
Policy: Import Quotas



Policy: Import Quotas

	Free Trade (with no quota)	With Quota		Impact of Quota	
		Trade Prohibition (quota = 0)	Quota = 3 Million Units per year	Impact of Trade Prohibition	Impact of Quota = 3 Million Units per year
Consumer Surplus	A + B + C + E + F + G + H + J + K	A	A + B + C + E	- B - C - E - F - G - H - J – K	- F - G - H - J - K
Producer Surplus	L	B + F + L	F + L	B + F	F
Net Benefits	A + B + C + E + F + G + H + J + K + L	A + B + F + L	A + B + C + E + F + L	- C - E - G - H - J - K	- G - H - J - K
Deadweight Loss	Zero	C + E + G + H + J + K	G + H + J + K	C + E + G + H + J + K	G + H + J + K
Producer Surplus (foreign)	Zero	Zero	H + J	Zero	H + J

Policy: Import Tariffs



Policy: Import Tariffs

	Free Trade (with no tariff)	With Tariff	Impact of Tariff
Consumer Surplus	A + B + C + E + F + G + H + J + K	A + B + C + E	- F - G - H - J - K
Producer Surplus	L	F + L	F
Impact on Government Budget	Zero	H + J	H + J
Net Benefits	A + B + C + E + F + G + H + J + K + L	A + B + C + E + F + L	- G - H - J – K
Deadweight Loss	Zero	G + K	G + K
Producer Surplus (foreign)	Zero	Zero	Zero

Comparing a Tariff to a Quota

Comparing a Tariff to a Quota

Let quota limit imports to Q3-Q2 . . . the equilibrium price would be the same as for the tariff . . . and the (world) deadweight loss would be the same as well.

Is there a difference? The quota generates no government revenue. Hence, while the total supply and total price for the domestic market remains the same under the two policies, *domestic* deadweight loss is larger under the quota.

Monopoly & Monopsony

Adapted from Chapter 11 of Besanko's Microeconomics

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June 25, 2025

Overview

1. The Monopolist's Profit Maximization Problem

2. The Welfare Economics and Monopoly

A Monopoly

Monopoly

A **Monopoly Market** consists of a single seller facing many buyers.

The monopolist's profit maximization problem:

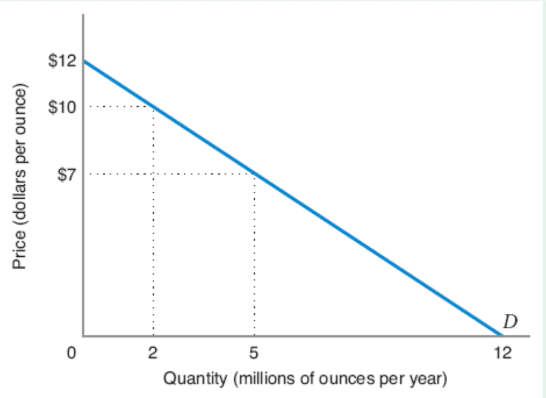
$$\max \pi(Q) = TR(Q) - TC(Q)Q$$

where $TR(Q) = QP(Q)$ and $P(Q)$ is the (inverse) market demand curve.
The monopolist's profit maximization condition:

$$\frac{\Delta TR(Q)}{\Delta Q} = \frac{\Delta TC(Q)}{\Delta Q}$$

$$MR(Q) = MC(Q)$$

A Monopoly - Profit Maximizing

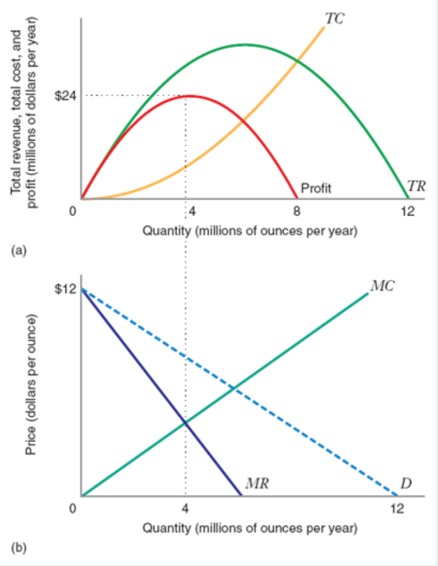


- Along the demand curve, different revenues for different quantities
- Profit maximization problem is the optimal trade-off between volume (number of units sold) and margin (the differential between price).

A Monopoly - Profit Maximizing

- Demand Curve: $P(Q) = 12 - Q$
- Total Revenue: $TR(Q) = Q \times P(Q) = 12Q - Q^2$
- Total Cost (Given): $TC(Q) = \frac{1}{2}Q^2$
- Profit-Maximization: $MR = MC$

A Monopoly - Profit Maximizing

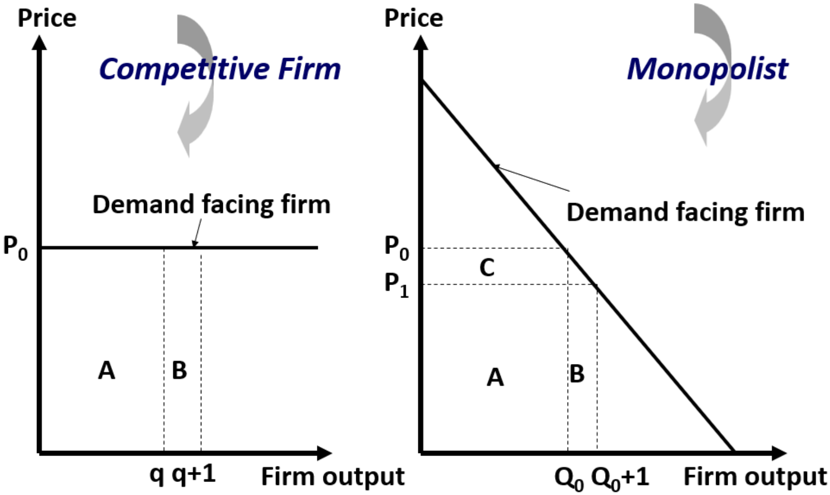


- As Q increases TC increases, TR increases first and then decreases.
- Profit Maximization is at $MR = MC$.

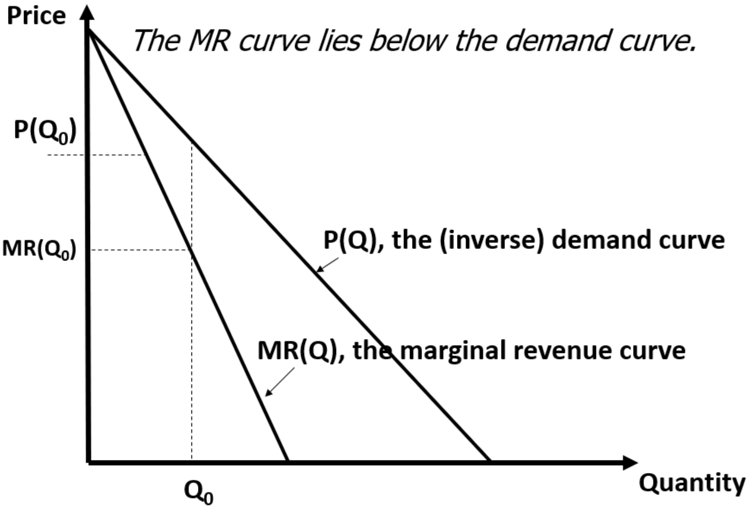
A Monopoly - Profit Maximizing

- $MR > MC$, firm can increase Q and increase profit.
- $MR < MC$, firm can decrease quantity and increase profit.
- $MR = MC$, firm cannot increase profit.
- Profit Maximizing Q: $MR(Q^*) = MC(Q^*)$

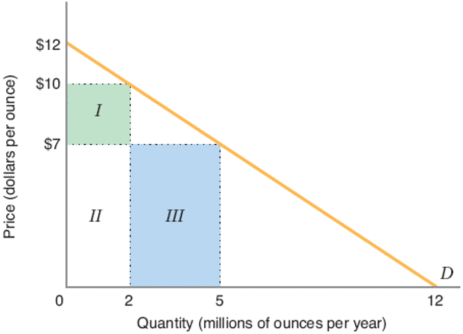
Marginal Revenue



Marginal Revenue Curve and Demand



Marginal Revenue Curve and Demand



- To sell more units, a monopolist has to lower the price.
- Increase in profit is Area *III* while revenue sacrificed at a higher price is Area *I*.
- Change in *TR* equals area *III* – area *I*.

Marginal Revenue Curve and Demand

- Area III = price $\times$ change in quantity $= P(\Delta Q)$
- Area I = - quantity $\times$ change in price $= -Q(\Delta P)$
- Change in monopolist profit: $P(\Delta Q) + Q(\Delta P)$

$$MR = \frac{\Delta TR}{\Delta Q} = \frac{P\Delta Q + Q\Delta P}{\Delta Q} = P + Q\frac{\Delta P}{\Delta Q}$$

Marginal Revenue

Marginal revenue has two parts:

- P: increase in revenue due to higher volume-the marginal units
- $Q\left(\frac{\Delta P}{\Delta Q}\right)$: decrease in revenue due to reduced price of the inframarginal units.
- The marginal revenue is less than the price the monopolist can charge to sell that quantity for any $Q > 0$.

Average Revenue

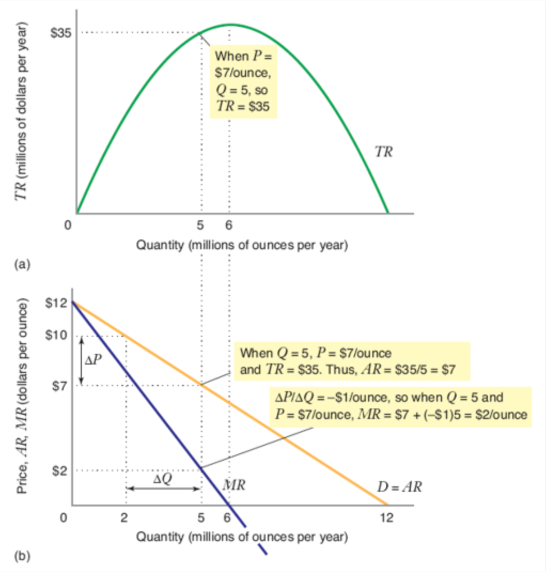
Since

$$AR = \frac{TR}{Q} = \frac{P \times Q}{Q} = P$$

The price a monopolist can charge to sell quantity Q is determined by the market demand curve the monopolists' average revenue curve is the market demand curve.

$$AR(Q) = P(Q)$$

Marginal Revenue and Average Revenue



- The demand curve D and average revenue curve AR coincide
- The marginal revenue curve MR lies below the demand curve

Marginal Revenue and Average Revenue

$$\frac{\Delta P}{\Delta Q} = -1$$

$$TR = P \times Q = 7 \times 5 = \$35 \text{ million per year}$$

$$AR = \frac{TR}{Q} = \frac{35}{5} = \$7 \text{ per ounce}$$

$$MR = P + Q\frac{\Delta P}{\Delta Q} = 7 + 5(-1) = \$2 \text{ per ounce}$$

When P decreases by \$3 per ounce, (from \$10 to \$7), quantity increases by 3 million ounces (from 2 million to 5 million per year).

Marginal Revenue and Average Revenue

Conclusions if $Q > 0$:

- $MR < P$
- $MR < AR$
- MR lies below the demand curve.

Marginal Revenue and Average Revenue

Given the demand curve, what are the average and marginal revenue curves?

$$P = a - bQ$$

$$AR = a - bQ$$

$$MR(Q) = P + \frac{\Delta P}{\Delta Q} Q$$

$$\frac{\Delta P}{\Delta Q} = -b$$

$$MR = a - bQ + Q(-b) = a - 2b$$

Vertical intercept is a.
Horizontal intercept is $Q = \frac{a}{2b}$

Profit Maximization

Given the inverse demand and MC, what is the profit-maximizing Q and P for the monopolist?

$$P = 12 - Q$$

$$MC = Q$$

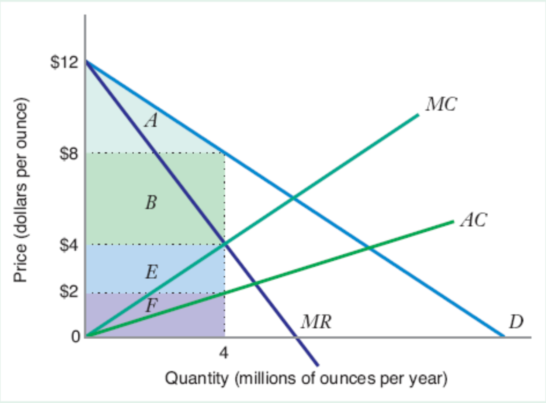
$$\text{Here } a = 12, b = 1$$

$$MR = 12 - 2Q$$

$$Q = 4$$

$$P = 12 - 4 = 8$$

Profit Maximization



- Profit Maximizing output is at $MR = MC$
- Monopolist will make 4 million ounces and sells at \$8 per ounce
- $TR =$ Areas $B + E + F$
- Profit $(TR - TC)$ is $B + E$
- Consumer surplus is area A

Shutdown Condition

In the short run, the monopolist shuts down if the most profitable price does not cover AVC. In the long run, the monopolist shuts down if the most profitable price does not cover AC. Here, P^* exceeds both AVC and AC.

Positive Profits for Monopolist

This profit is positive. *Why? Because the monopolist takes into account the price-reducing effect of increased output so that the monopolist has less incentive to increase output than the perfect competitor.*

Profit can remain positive in the long run. *Why? Because we are assuming that there is no possible entry in this industry, so profits are not competed away.*

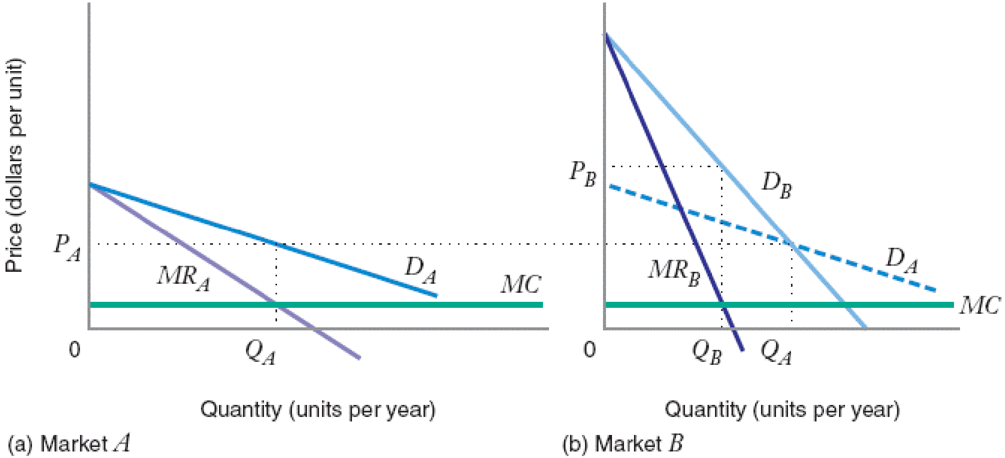
Equilibrium

A monopolist does not have a supply curve (i.e., an optimal output for any exogenously-given price) because price is *endogenously-determined* by demand: the monopolist picks a preferred point on the demand curve.

One could also think of the monopolist choosing output to maximize profits subject to the *constraint* that price be determined by the demand curve.

Price Elasticity of Demand

- Market A profit maximizing price is P_A.
- Market B profit maximizing price is P_B. Demand is less elastic in Market B.



Inverse Elasticity Pricing Rule

We can rewrite the MR curve as follows:

$$MR = P + Q\left(\frac{\Delta P}{\Delta Q}\right) =$$

$$= P\left(1 + \left(\frac{Q}{P}\right)\left(\frac{\Delta P}{\Delta Q}\right)\right) =$$

$$= P\left(1 + \frac{1}{\varepsilon}\right)$$

where: ε is the price elasticity of demand, $\left(\frac{P}{Q}\right)\left(\frac{\Delta Q}{\Delta P}\right)$

- When demand is elastic ($\varepsilon < -1$), $MR > 0$
- When demand is inelastic ($\varepsilon > -1$), $MR < 0$
- When demand is unit elastic ($\varepsilon = -1$), $MR = 0$

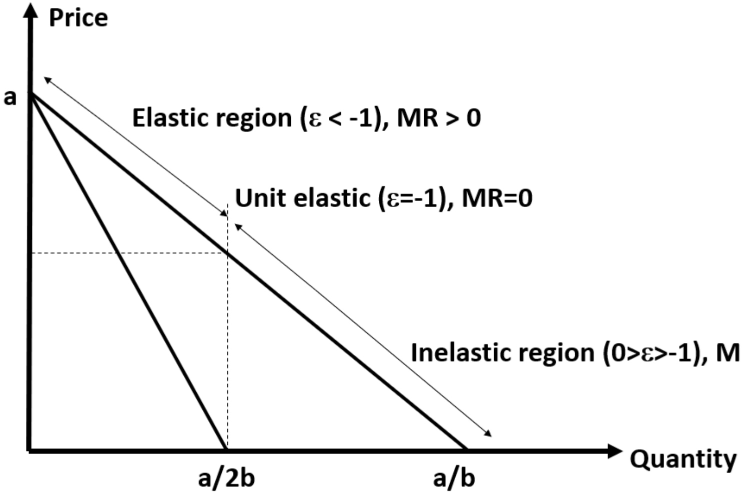
Inverse Elasticity Pricing Rule

Given the constant elasticity demand curve and MC:

- What is the optimal P when $Q = 100P^{-2}$?
- What is the optimal P when $Q = 100P^{-5}$?

$$Q = aP^{-b}$$
$$\text{Price elasticity of demand } = -b$$
$$MC = \$50$$
$$\text{for } Q = 100P^{-2}$$
$$\text{Price elasticity of demand } \varepsilon_{Q,P} = -2$$
$$\frac{P-50}{P} = -\frac{1}{-2} \Rightarrow P = \$100$$
$$\text{for } Q = 100P^{-5}$$
$$\text{Price elasticity of demand } \varepsilon_{Q,P} = -5$$
$$\frac{P-100}{P} = -\frac{1}{-5} \Rightarrow P = \$62.50$$

Elasticity Region of the Linear Demand Curve



Marginal Cost and Price Elasticity Demand

- Profit maximizing condition is $MR = MC$ with P^* and Q^*:

$$MR(Q^*) = MC(Q^*)$$

$$MC(Q^*) = P^* \left(1 + \frac{1}{\varepsilon_{Q,P}}\right)$$

- Rearranging and setting $MR(Q^*) = MC(Q^*)$

$$\frac{P^* - MC^*}{P^*} = -\frac{1}{\varepsilon_{Q,P}}$$

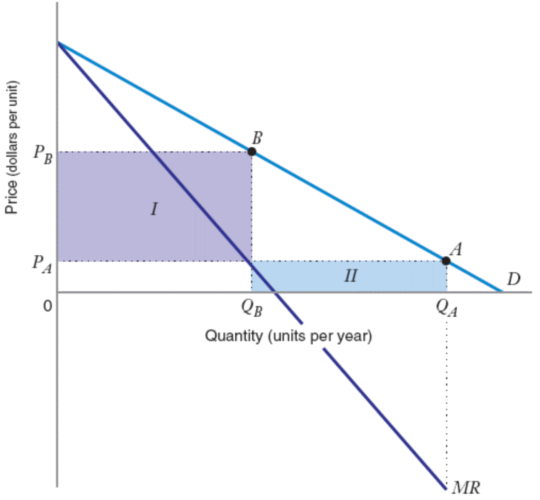
Inverse Elasticity Pricing Rule

Inverse Elasticity Pricing Rule

Inverse Elasticity Pricing Rule: Monopolist's optimal markup of price above marginal cost expressed as a percentage of price is equal to minus the inverse of the price elasticity of demand.

$$\frac{P^* - MC^*}{P^*} = -\frac{1}{\varepsilon_{Q,P}}$$

Price Elasticity



Monopolist operates at the elastic region of the market demand curve. Increasing price from *PA* to *PB*, *TR* increases by area *I* – area *II* and total cost goes down because monopolist is producing less.

Elasticity Region of the Demand Curve

Therefore, the monopolist will always operate on the elastic region of the market demand curve. As demand becomes more elastic at each point, marginal revenue approaches price.

Elasticity Region of the Demand Curve

Example

Now, suppose that $Q_D = 100P^{-b}$ and $MC = c$ (constant). What is the monopolist's optimal price now?

$$P\left(1 + \frac{1}{-b}\right) = c$$

$$P^* = \frac{cb}{(b-1)}$$

We need the assumption that $b > 1$ (*demand is everywhere elastic*) to get an interior solution.

As $b \to 1$ (*demand becomes everywhere less elastic*), $P^* \to \infty$ and $P - MC$, the *price-cost margin* also increases to infinity.

As $b \to \infty$, the monopoly price approaches marginal cost.

Market Power

Market Power

An agent has **Market Power** if s/he can affect, through his/her own actions, the price that prevails in the market. Sometimes this is thought of as the degree to which a firm can raise price above marginal cost.

The Lerner Index of Market Power

The Lerner Index of Market Power

The **Lerner Index of market power** is the price-cost margin, $\frac{(P^* - MC)}{P^*}$. This index ranges between 0 (for the competitive firm) and 1, for a monopolist facing a unit elastic demand.

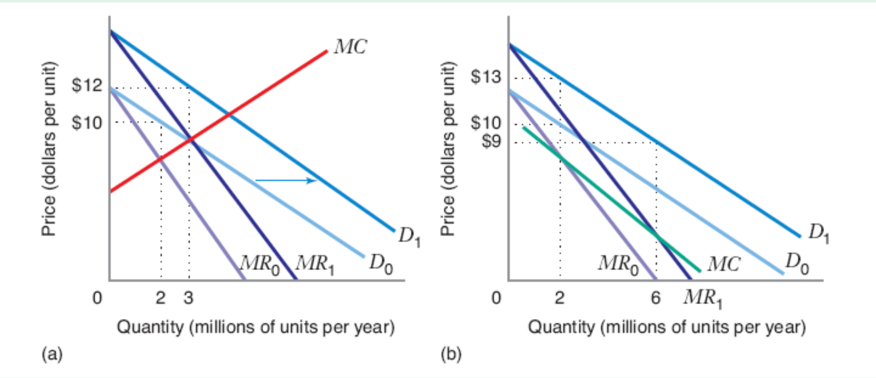
Restating the monopolist's profit maximization condition, we have:

$$P^* \left(1 + \frac{1}{\varepsilon}\right) = MC(Q^*)$$

$$\frac{[P^* - MC(Q^*)]}{P^*} = -\frac{1}{\varepsilon}$$

In words, the monopolist's ability to price above marginal cost depends on the elasticity of demand.

Comparative Statics - Shifts in Market Demand

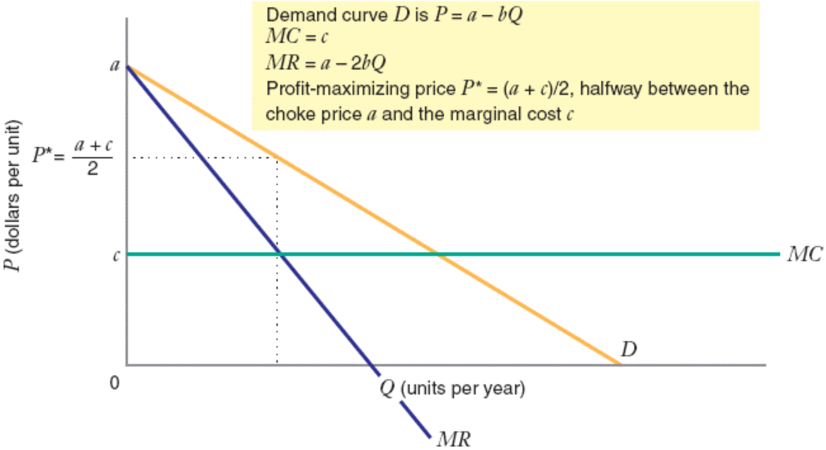


Rightward shift in the demand curve causes an increase in profit maximizing quantity.

- MC is increases as Q increases
- MC decreases as Q increases

Comparative Statics – Monopoly Midpoint Rule

For a constant MC, profit maximizing price is found using the monopoly midpoint rule – The optimal price P^* is halfway between the vertical intercept of the demand curve a (choke price) and vertical intercept of the MC curve c.



Comparative Statics – Monopoly Midpoint Rule

Given P and MC what is the profit maximizing P and Q?

$$P = a - bQ$$

$$MC = c$$

$$MR = a - 2bQ$$

$$MR = MC$$

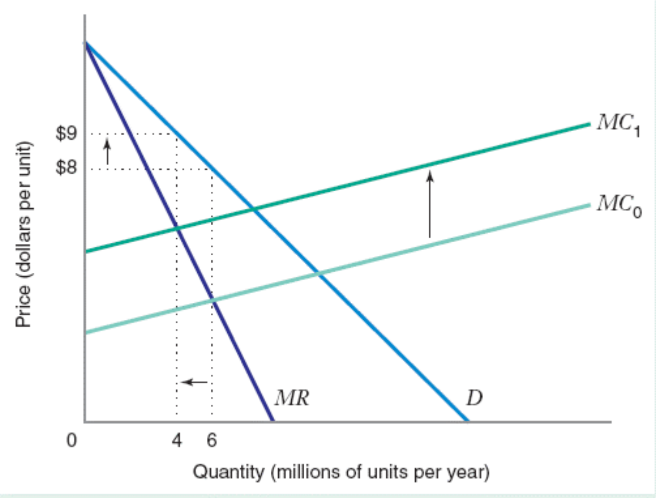
$$a - 2bQ^* = c$$

$$Q^* = \frac{a-c}{2b}$$

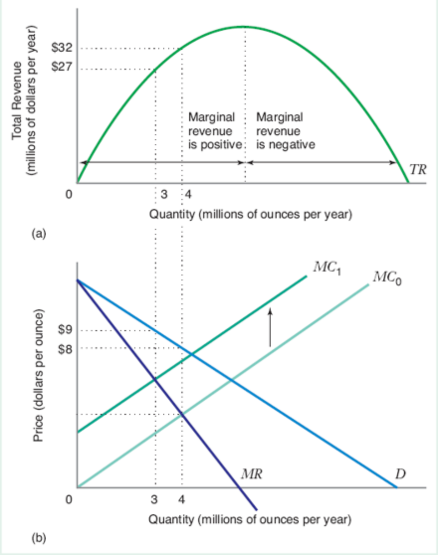
$$P^* = a - b\left(\frac{1-c}{2b}\right) = a - \frac{1}{2}a + \frac{1}{2}c = \frac{a+c}{2}$$

Comparative Statics – Shifts in Marginal Cost

When MC shifts up, Q falls and P increases.



Comparative Statics – Revenue and MC shifts



- Upward shift of MC decreases the profit maximizing monopolist's total revenue.
- Downward shift of MC increases the profit maximizing monopolist's total revenue.

Cartel

Cartel

A **cartel** is a group of firms that collusively determine the price and output in a market. In other words, a cartel acts as a single monopoly firm that maximizes total industry profit.

Cartel

The problem of optimally allocating output across cartel members is identical to the monopolist's problem of allocating output across individual plants.

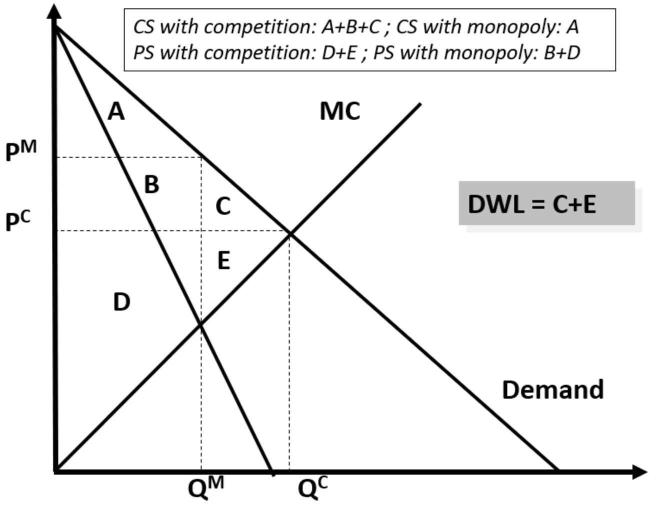
Therefore, a cartel does not necessarily divide up market shares equally among members: higher marginal cost firms produce less.

This gives us a benchmark against which we can compare actual industry and firm output to see how far the industry is from the collusive equilibrium

The Welfare Economies of Monopoly

Since the monopoly equilibrium output does not, in general, correspond to the perfectly competitive equilibrium it entails a dead-weight loss.
Suppose that we compare a monopolist to a competitive market, where the supply curve of the competitors is equal to the marginal cost curve of the monopolist.

The Welfare Economies of Monopoly



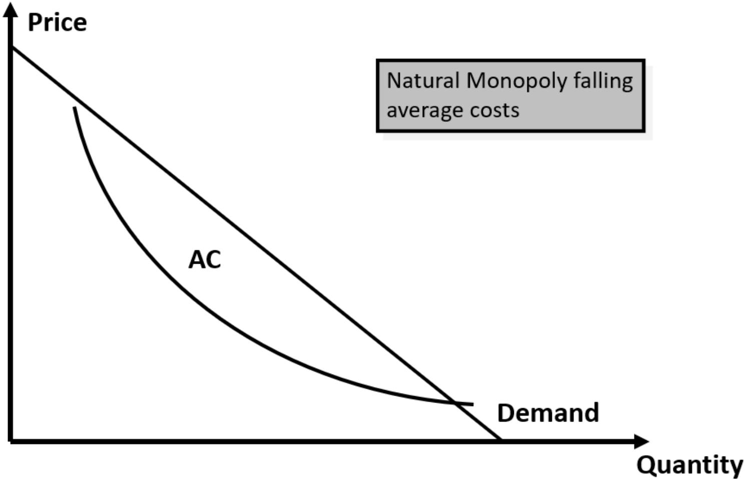
Natural Monopolies

Natural Monopoly

A market is a **natural monopoly** if the total cost incurred by a single firm producing output is less than the combined total cost of two or more firms producing this same level of output among them.

Benchmark: Produce where $P = AC$

Natural Monopolies



Barriers to Entry

Barriers to Entry

Factors that allow an incumbent firm to earn positive economic profits while making it unprofitable for newcomers to enter the industry.

1. **Structural Barriers to Entry** – occur when incumbent firms have cost or demand advantages that would make it unattractive for a new firm to enter the industry
2. **Legal Barriers to Entry** – exist when an incumbent firm is legally protected against competition
3. **Strategic Barriers to Entry** – result when an incumbent firm takes explicit steps to deter entry

A Monopsony

Monopsony

A **Monopsony Market** consists of a single buyer facing many sellers.

The monopsonist's profit maximization problem:

$$\max \pi = TR - TC = P^* f(L) - w^* L$$

where $Pf(L)$ is the total revenue for monopsonist and $w^* L$ is the total cost.
The monopsonist's profit maximization condition:

$$MRP_L = P^* MP_L = P\left(\frac{\Delta Q}{\Delta L}\right) = \frac{\Delta TC}{\Delta L} = w + L\left(\frac{\Delta w}{\Delta L}\right) = ME_L$$

Monopsony

Example

$$Q = 5L$$

$$P = \$10 \text{ per unit}$$

$$w = 2 + 2L$$

$$ME_L = w + L\left(\frac{\Delta w}{\Delta L}\right) = 2 + 4L$$

$$MRP_L = P^*\left(\frac{\Delta Q}{\Delta L}\right) = 10 \times 5 = 50$$

$$MRP_L = ME_L$$

$$2 + 4L = 50 \Rightarrow L = 12$$

$$w = 2 + 2L = \$26$$

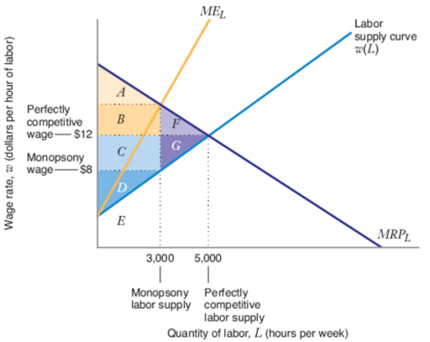
Inverse Elasticity Pricing Rule

Monopsony equilibrium condition results in:

$$\frac{MRP_L - w}{w} = \frac{1}{\varepsilon_{L,w}}$$

where: ε is the price elasticity of labor supply, $\frac{w}{L}\frac{\Delta L}{\Delta w}$

The Welfare Economies of Monopsony



	Perfect Competition	Monopsony	Impact of Monopsony
Consumer surplus	$A + B + F$	$A + B + C$	$C - F$
Producer surplus	$C + D + G$	D	$-C - G$
Net economic benefit	$A + B + C + D + F + G$	$A + B + C + D$	$-F - G$